

DETERMINISTIC TRUTHFUL  
APPROXIMATION MECHANISMS FOR  
SCHEDULING RELATED MACHINES

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# INTERNET

- NO CENTRAL AUTHORITY
- SELF-INTERESTED COMPONENTS

- NOT ALTRUISTIC, NOR MALICIOUS
- DIFFERENT GOALS
- MAY NOT FOLLOW THE "PROTOCOL"

USERS  
PROVIDERS  
AUTONOMOUS  
SYSTEMS  
PRIVATE  
COMPANIES  
UNIVERSITIES  
⋮

# INTERNET

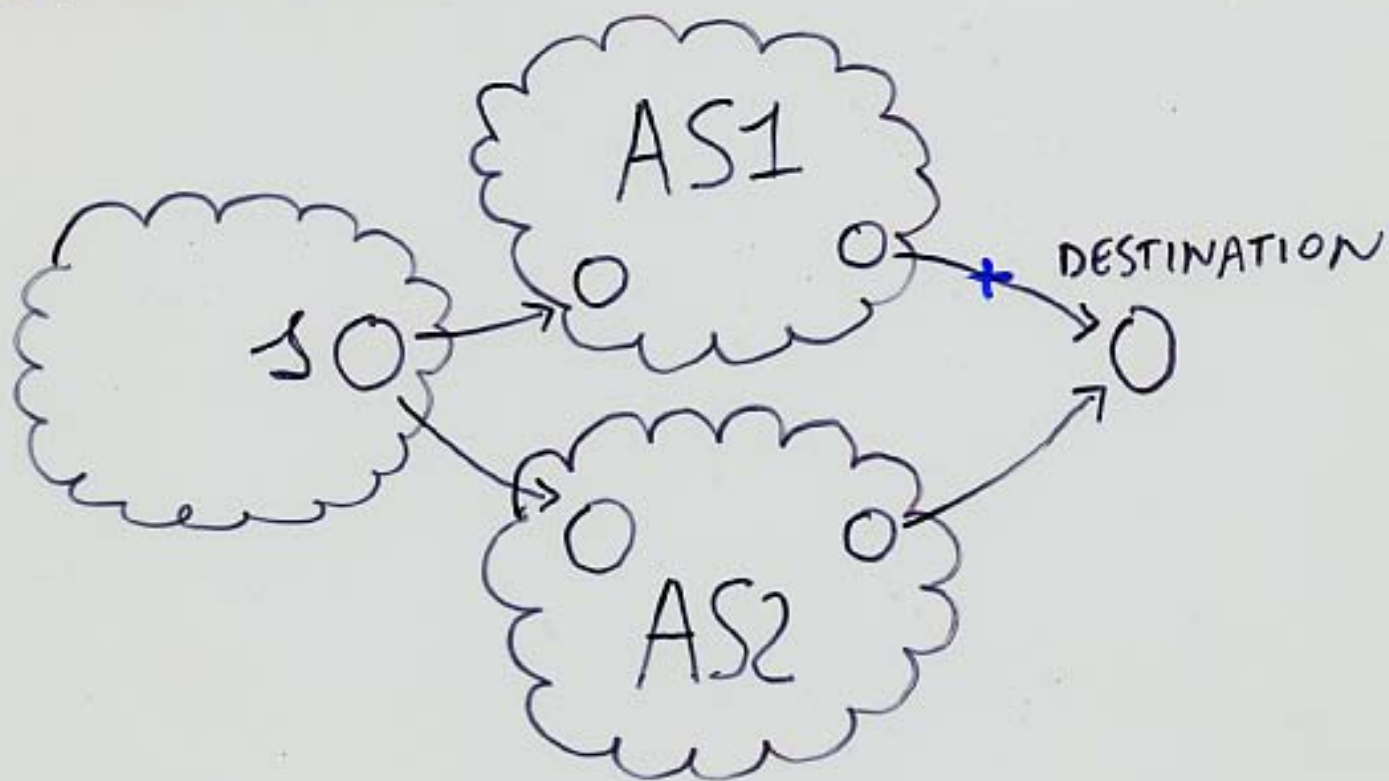
- NO CENTRAL AUTHORITY

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- NOT ALTRUISTIC, NOR MALICIOUS
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SELFISH  $\updownarrow$  AGENTS  
(RATIONAL PLAYERS)

# AUTONOMOUS SYSTEMS:



FALSE LINK STATUS



REDIRECT TRAFFIC



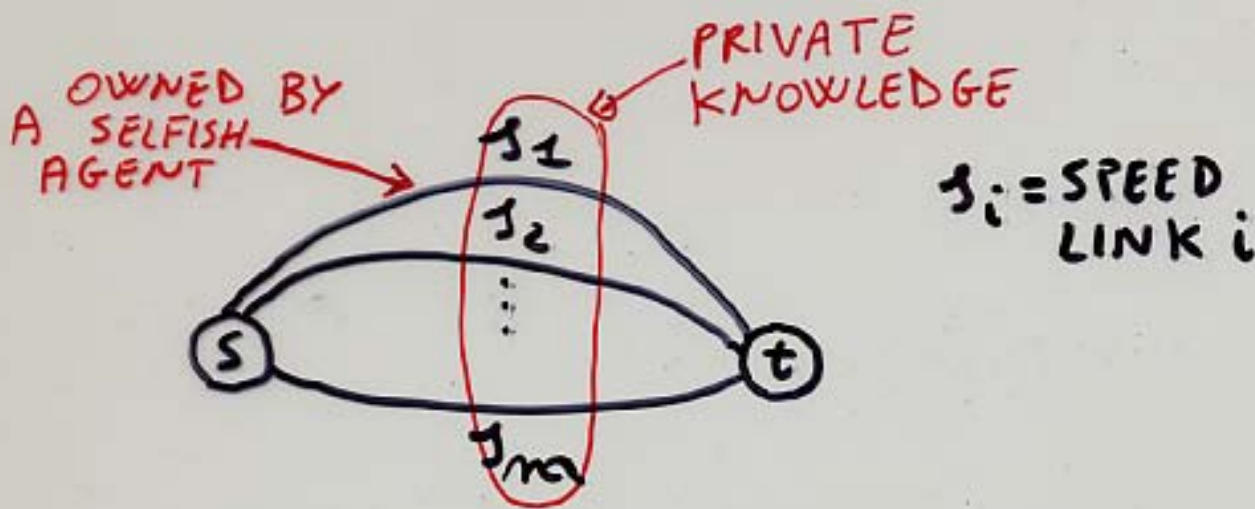
$s_i = \text{SPEED}$   
 $\text{LINK } i$

UNSPLITTABLE TRAFFIC  $t_1, \dots, t_m$

GOAL: MINIMIZE THE MAKESPAN

$$\text{MAX}_{1 \leq i \leq m} \left\{ \frac{w_i}{s_i} \right\}$$

$$w_i = \sum_{\substack{j \text{ ASSIGNED} \\ \text{TO LINK } i}} t_j$$



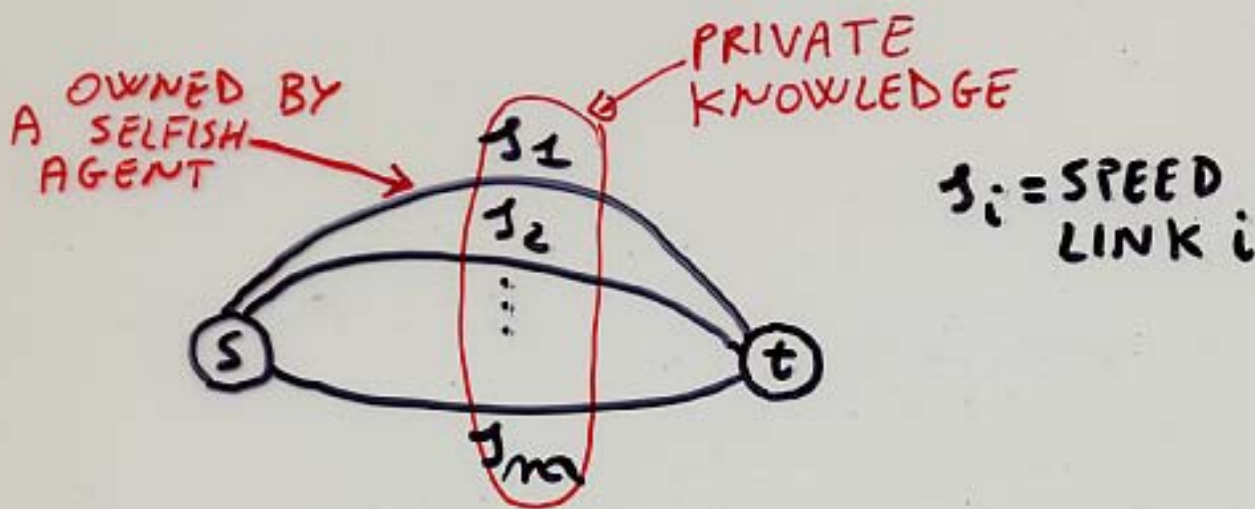
UNSPPLITTABLE TRAFFIC  $t_1, \dots, t_m$

GOAL: MINIMIZE THE MAKESPAN

$$\text{MAX}_{1 \leq i \leq m} \left\{ \frac{w_i}{\beta_i} \right\}$$

COST FOR AGENT  $i$   
(ONE AGENT PER LINK)

$$w_i = \sum_{j \text{ ASSIGNED TO LINK } i} t_j$$



UNSPPLITTABLE TRAFFIC  $t_1, \dots, t_m$

GOAL: MINIMIZE THE MAKESPAN

$$\text{MAX}_{1 \leq i \leq m} \left\{ \frac{w_i}{\beta_i} \right\}$$

COST FOR AGENT  $i$   
(ONE AGENT PER LINK)

$$w_i = \sum_{j \text{ ASSIGNED TO LINK } i} t_j$$

AGENTS MAY LIE (REPORT  $\tau_i \neq \beta_i$ )

IDEA: PAY THE AGENTS TO INDUCE TRUTHFUL BEHAVIOR ( $\tau_i = \beta_i$ )

# MECHANISMS

$$M = (\text{ALG}, P)$$

$$\text{ALG}(\pi_1, \dots, \pi_i, \dots, \pi_m)$$

$$P_i(\pi_1, \dots, \pi_i, \dots, \pi_m)$$



$$\text{COST}_i = \frac{w_i^{\text{ALG}}(\pi_1, \dots, \pi_i, \dots, \pi_m)}{\delta_i}$$

UTILITY (NET PROFIT):

$$U_i(\pi_1, \dots, \pi_i, \dots, \pi_m) = P_i(\dots \pi_i \dots) - \text{COST}_i$$



CAN WE USE EVERY ALG?

NO!!

-  $M = (\text{ALG}, P)$  TRUTHFUL  $\Rightarrow$

ALG MUST BE MONOTONE

- ALG MONOTONE  $\Rightarrow \exists P_{\text{ALG}}$  s.t.

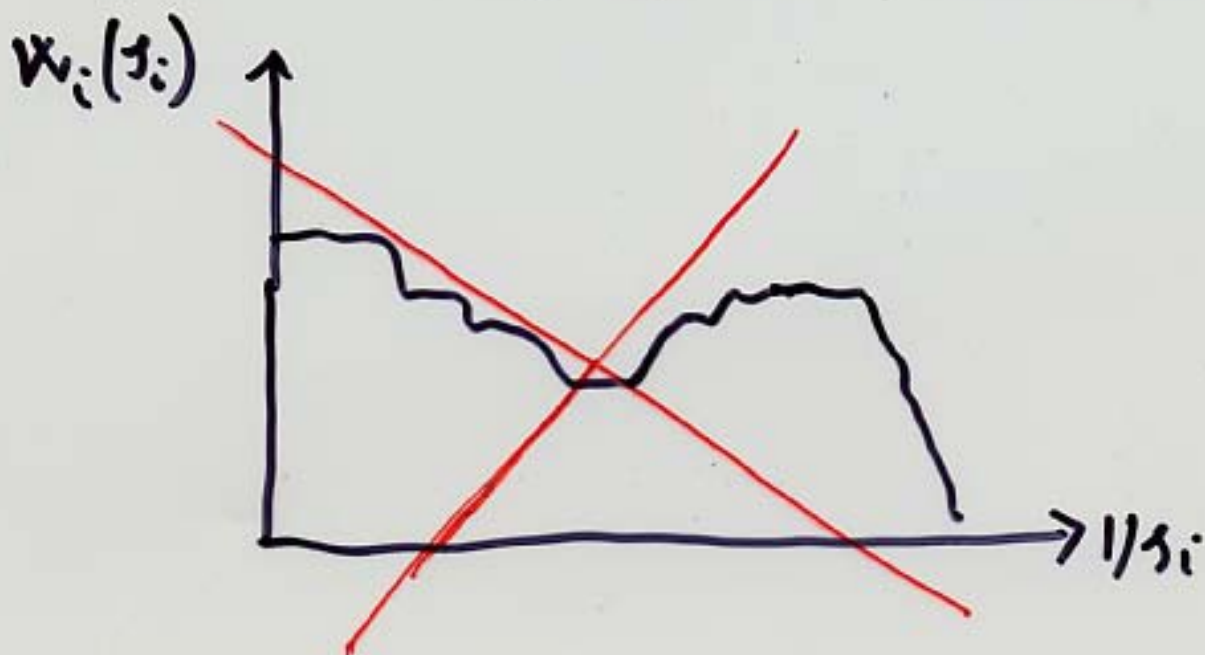
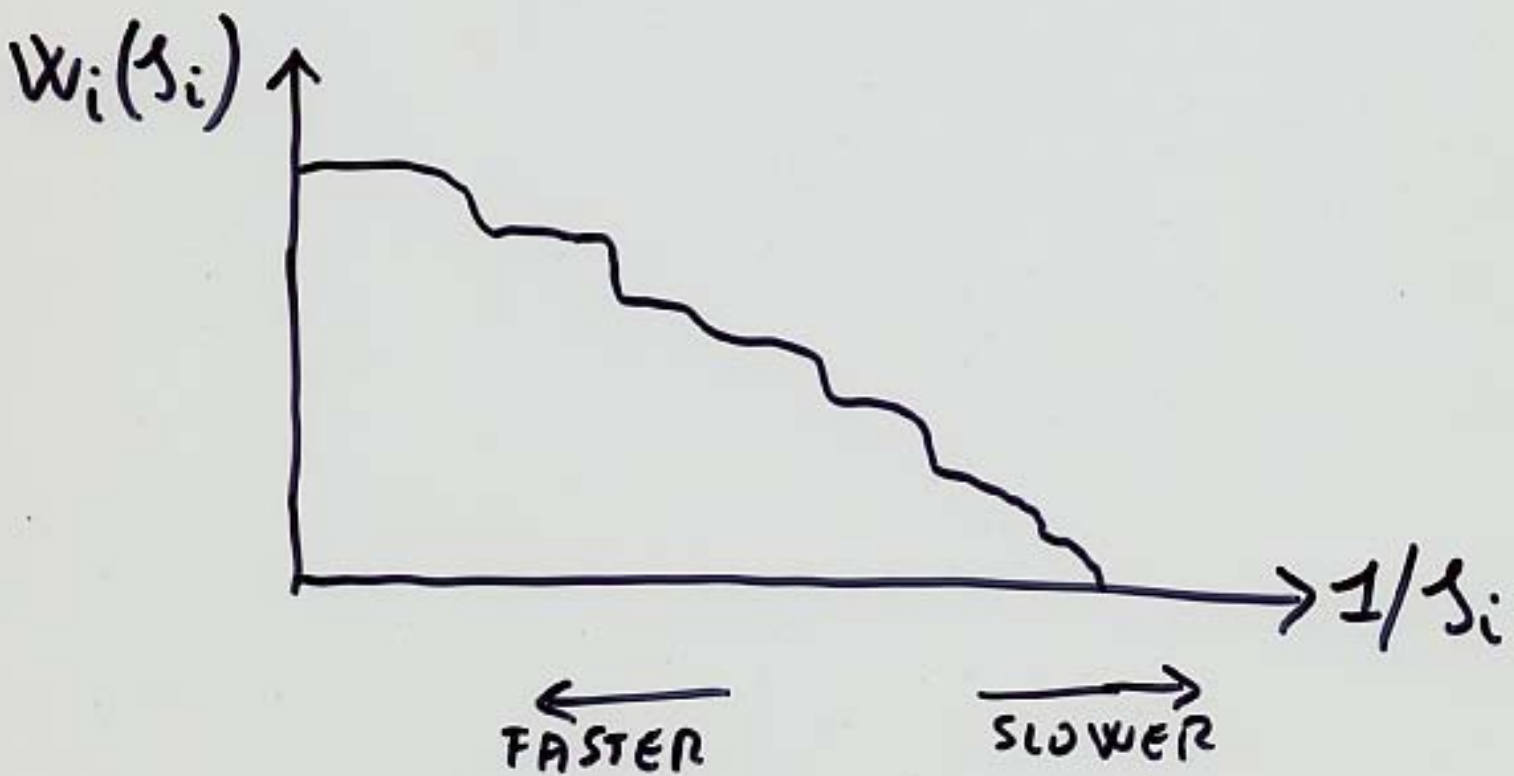
$(\text{ALG}, P_{\text{ALG}})$  IS TRUTHFUL

[ARCHER-TARDOS, 2001]

# MONOTONE ALGOS

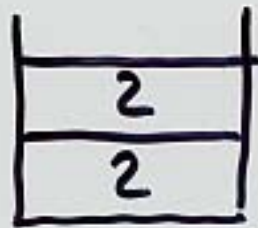
FIX  $\mathcal{I}_{-i} = \mathcal{I}_1, \dots, \mathcal{I}_{i-1}, \mathcal{I}_{i+1}, \dots, \mathcal{I}_m$

$$W_i(\mathcal{I}_i) = W_i^{\text{ALG}}(\mathcal{I}_1, \dots, \mathcal{I}_{i-1}, \mathcal{I}_i, \mathcal{I}_{i+1}, \dots, \mathcal{I}_m)$$

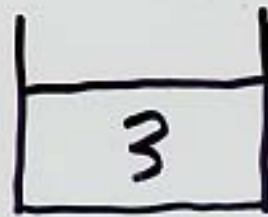


# GREEDY IS NOT MONOTONE

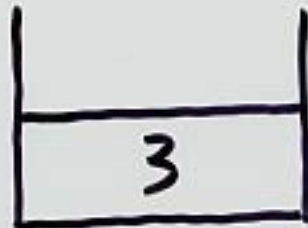
JOB: 3, 2, 2



$$s_1 = 1$$

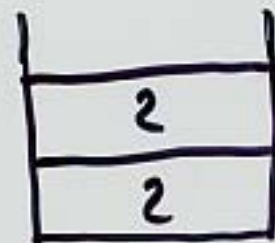


$$s_2 = 1 + \epsilon$$



$$s_1' = (1 + \epsilon)^2$$

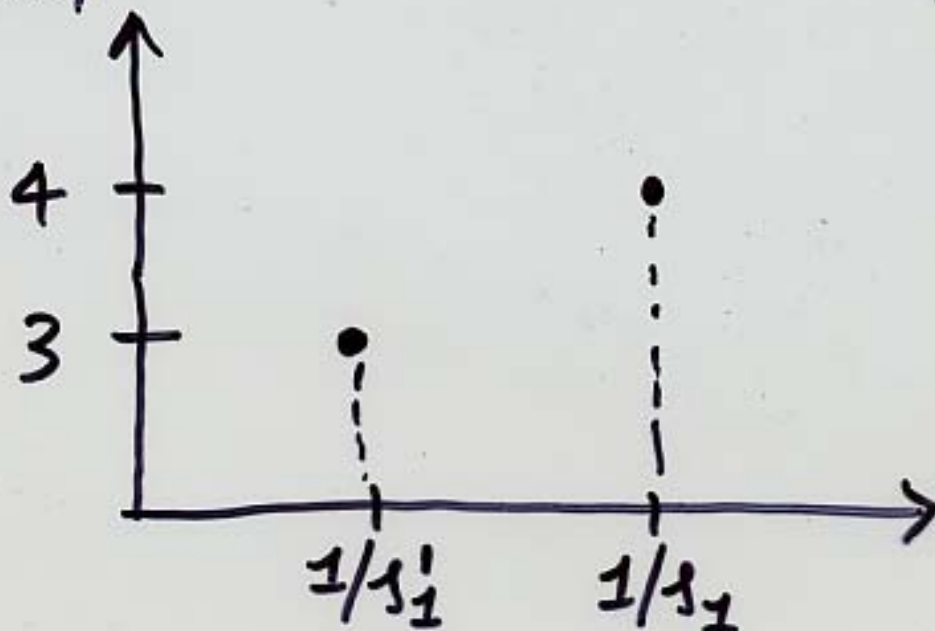
$$\Downarrow \\ 1 + \epsilon$$



$$s_2 = 1 + \epsilon$$

$$\Downarrow \\ 1$$

$W_1^{\text{GREEDY}}$



PROBLEM VERSION

MONOTONE  
APX ALGORITHMS

ANY  $m$

1 EXPTIME

[ARCHER-TARDOS 01]

$3+\epsilon$

POLYTIME

RANDOMIZED

TRUTHFUL IN EXPECTATION

$m \in O(1)$

$s_{\max} \in O(1)$

$2+\epsilon$

DETERMINISTIC  
POLYTIME

DIVISIBLE  
SPEEDS ( $s_i = 2^{c_i}$ )

$2+\epsilon$

"

[THIS WORK]

ANY SPEEDS

$4+\epsilon$

"

PROBLEM VERSION

MONOTONE  
APX ALGORITHMS

ANY  $m$

1 EXPTIME

[ARCHER-TARDOS 01]

$3+\epsilon$  POLYTIME  
RANDOMIZED

TRUTHFUL IN EXPECTATION

WEAKER NOTION

$2+\epsilon$  DETERMINISTIC  
POLYTIME

$2+\epsilon$  "

[THIS WORK]

$4+\epsilon$  "

DOMINANT STRATEGIES

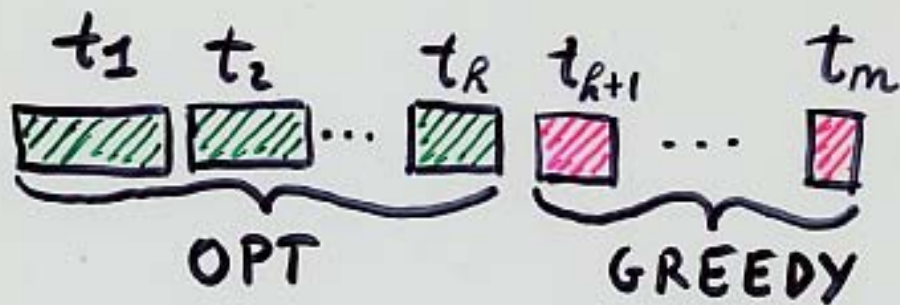
$m \in O(1)$

$s_{MAX} \in O(1)$

DIVISIBLE SPEEDS ( $s_i = 2^{c_i}$ )

ANY SPEEDS

# OVERVIEW



PTAS  
(1+ $\epsilon$ )-APX



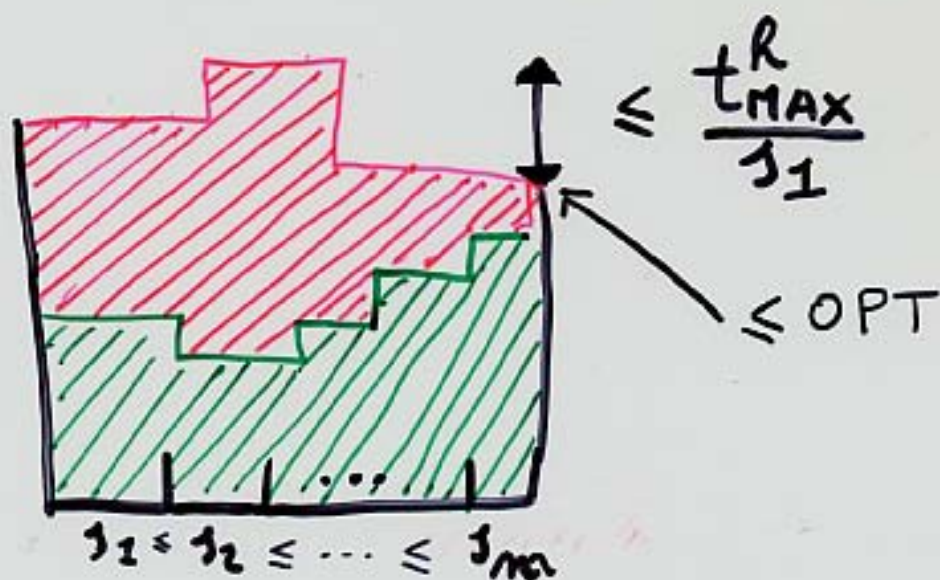
PTAS-INDEPENDENT

(2+ $\epsilon$ )-APX



PTAS-MONOTONE:

REPLACE GREEDY WITH A  
MONOTONE "SUFFICIENTLY GOOD"  
ALGORITHM



$$t_{MAX}^R = \max\{t_{R+1}, \dots, t_m\} \leq t_R$$

$$APX \leq OPT + \frac{t_R}{s_1} \qquad OPT \geq \frac{R \cdot t_R}{\sum s_i}$$

$$APX \leq OPT + \frac{OPT}{R \cdot s_1} \cdot \sum s_i$$

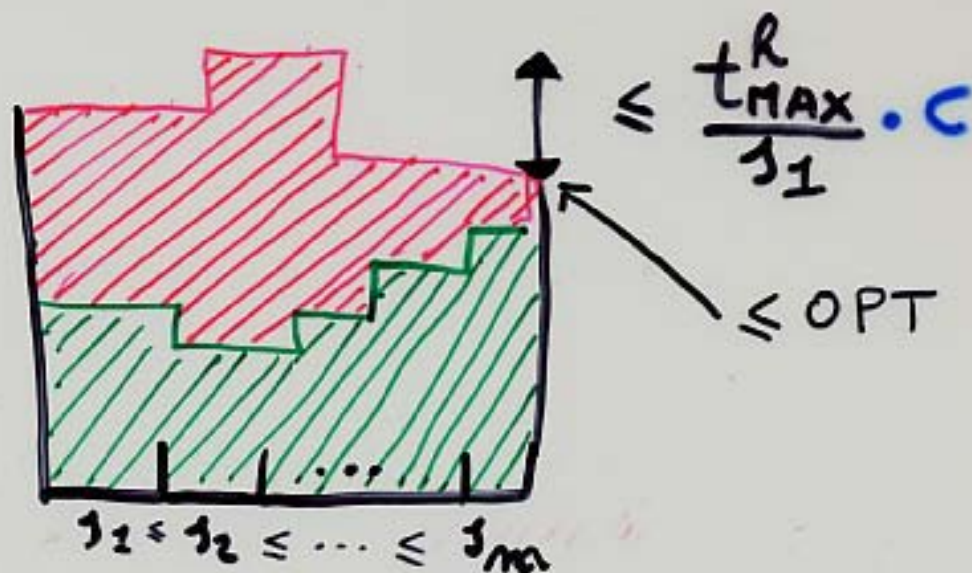
$$= OPT \left( 1 + \frac{\sum s_i}{R \cdot s_1} \right)$$

$$\leq OPT \left( 1 + \frac{m \cdot s_m}{s_1} \right)$$

$s_m \gg s_1$   
 $\Downarrow$   
 IGNORE MACHINE  
 $s_1$

$s_m \approx s_1$   
 $\Downarrow$   
 $m/R = \epsilon$

# C-GREEDY-CLOSE ALGO



$$t_{MAX}^R = \text{MAX} \{t_{R+1}, \dots, t_m\} \leq t_R$$

$$APX \leq OPT + \frac{t_R}{s_1} \cdot c \quad \text{OPT} \geq \frac{R \cdot t_R}{\sum s_i}$$

$$APX \leq OPT + \left( \frac{OPT}{R \cdot s_1} \cdot \sum s_i \right) \cdot c$$

$$= OPT \left( 1 + \frac{\sum s_i}{R \cdot s_1} \right) \cdot c$$

$$\leq OPT \left( 1 + \frac{m \cdot s_m}{s_1} \right) \cdot c$$

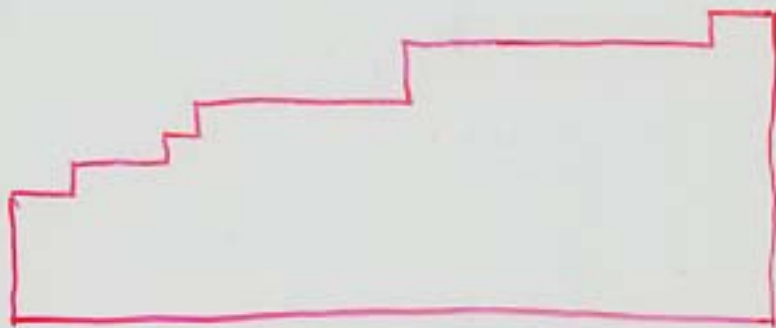
$s_m \gg s_1$   
 $\Downarrow$   
 IGNORE MACHINE  
 $s_1$

$s_m \approx s_1$   
 $\Downarrow$   
 $c \cdot m / R = \epsilon$



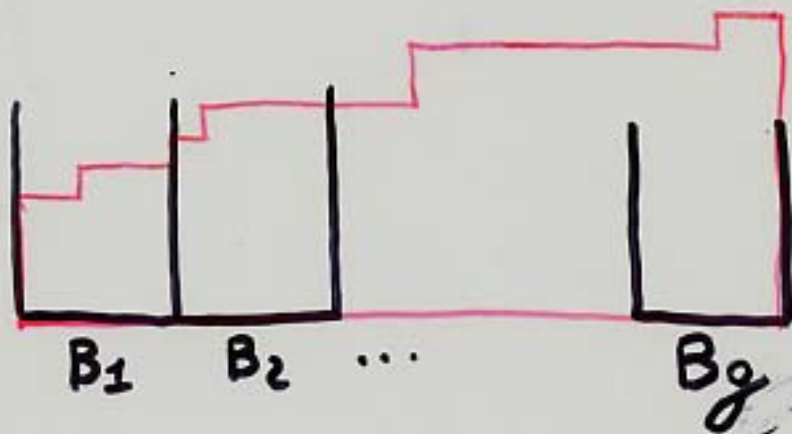
1) RUN GREEDY ON  $S = \sum a_i$   
IDENTICAL MACHINES

2) ORDER THE LOADS



1) RUN GREEDY ON  $S = \sum s_i$ :  
IDENTICAL MACHINES

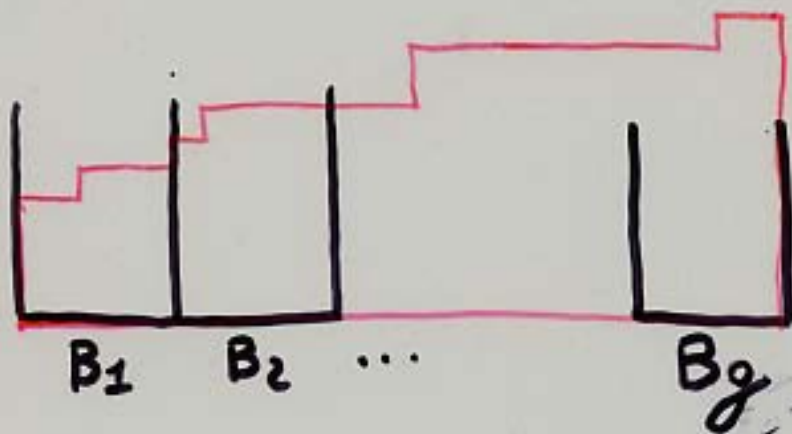
2) ORDER THE LOADS



3) CONSIDER  $g = \text{GCD}(s_1, \dots, s_m)$  BLOCKS

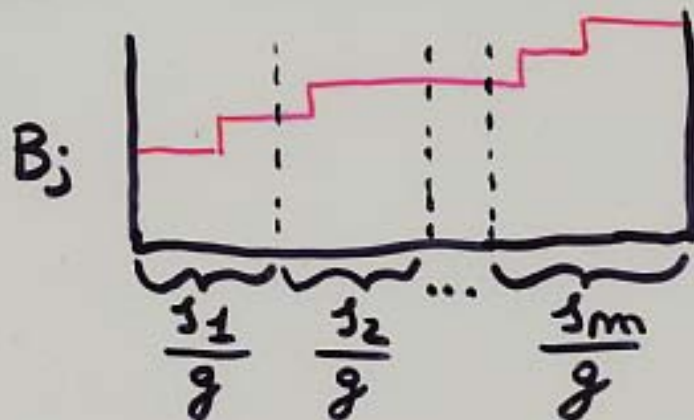
1) RUN GREEDY ON  $S = \sum s_i$ :  
IDENTICAL MACHINES

2) ORDER THE LOADS



3) CONSIDER  $g = \text{GCD}(s_1, \dots, s_m)$  BLOCKS

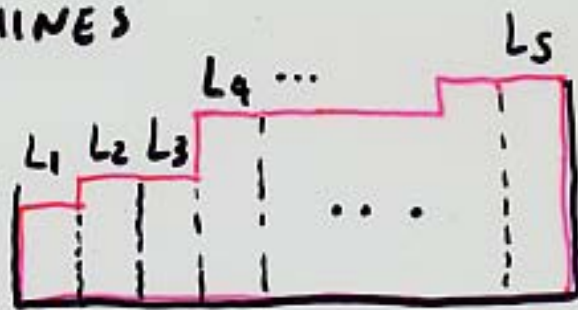
4) ASSIGN "BLOCKWISE"



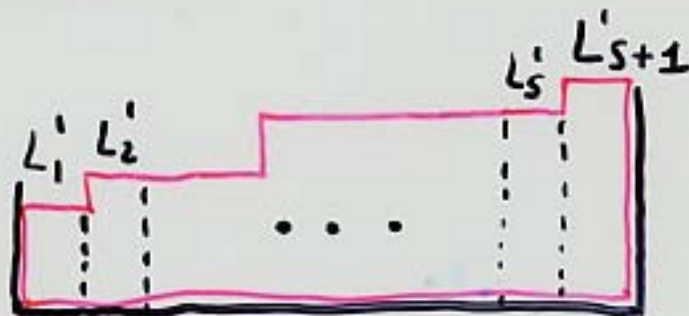
THEOREM:  $\text{COST} \leq \text{OPT} + \frac{t_{\text{MAX}}}{g}$

# MONOTONICITY

S MACHINES



S+1 MACHINES



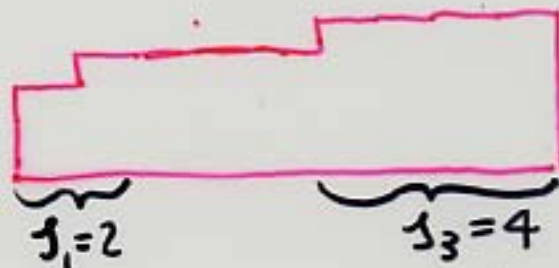
$$L'_i \leq L_i$$

$$L'_{S+1-i} \leq L_{S-i}$$

$$s_2 = 2$$

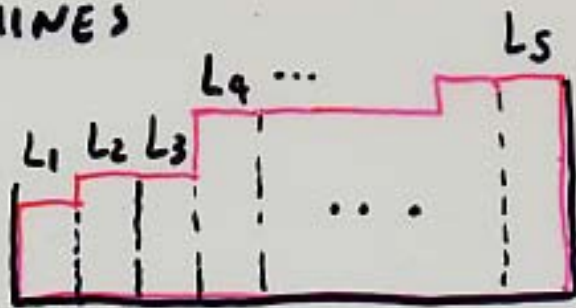


$$s'_2 = 3$$

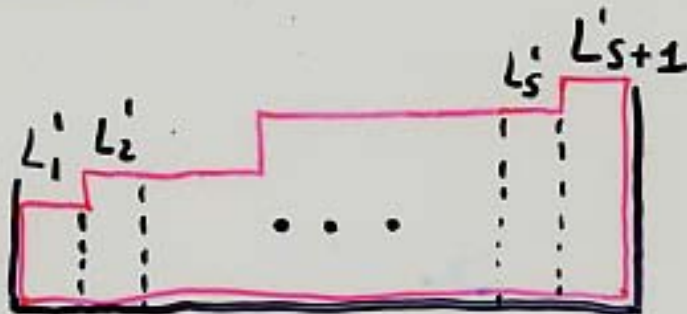


# MONOTONICITY

S MACHINES



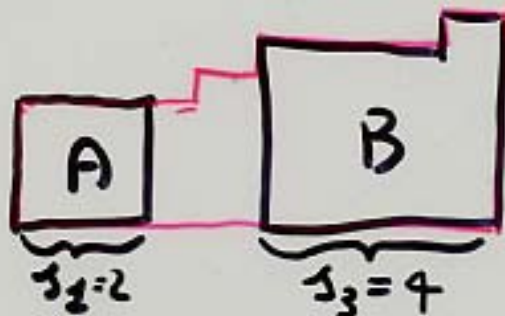
S+1 MACHINES



$$L'_i \leq L_i$$

$$L'_{s+1-i} \leq L_{s-i}$$

$$s_2 = 2$$



$$s'_2 = 3$$

