DETERMINISTIC TRUTHFUL APPROXIMATION MECHANISMS FOR SCHEDULING RELATED MACHINES

VINCENZO AULETTA, ROBERTO DE PRISCO,
PAOLO PENNA, PINO PERSIANO

UNIVERSITÀ DI SALERNO

INTERNET

- NO CENTRAL AUTHORITY
- SELF-INTERESTED COMPONENTS
  - NOT ALTRUISTIC, NOR MALICIOUS
  - DIFFERENT GOALS
  - MAY NOT FOLLOW THE "PROTOCOL"

USERS
PROVIDERS
AUTONOMOUS SYSTEMS
PRIVATE COMPANIES
UNIVERSITIES
⋮

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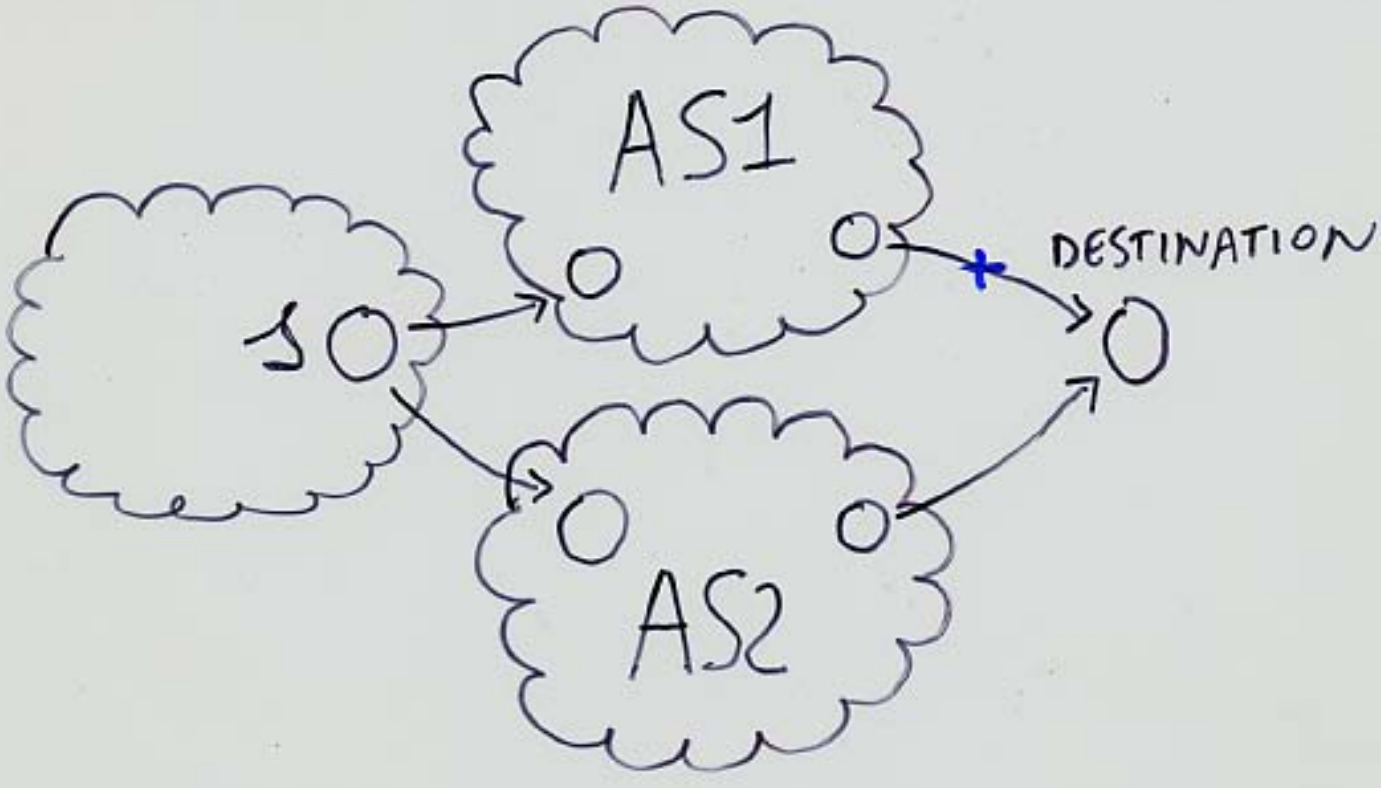
SELFISH AGENTS
(RATIONAL PLAYERS)

AUTONOMOUS SYSTEMS:



FALSE LINK STATUS

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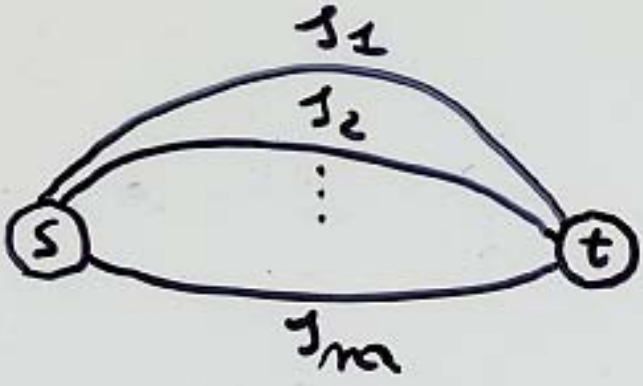
REDIRECT TRAFFIC



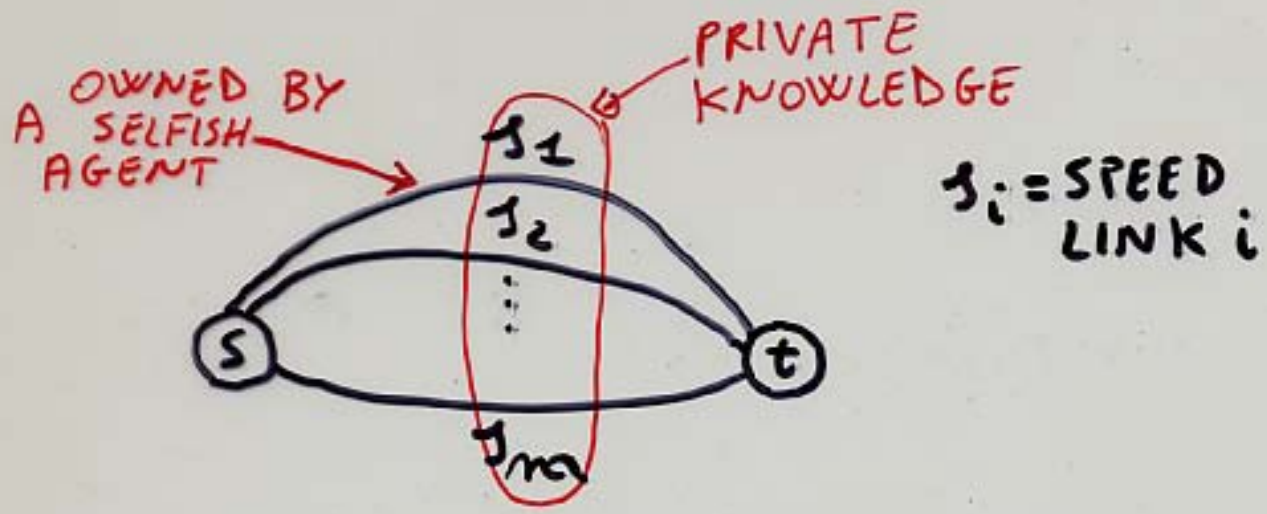
s_i = SPEED LINK i

UNSPLITTABLE TRAFFIC $t_1, \ldots, t_m$

GOAL: MINIMIZE THE MAKESPAN

$$\max_{1 \le i \le m} \left\{ \frac{w_i}{s_i} \right\}$$

$$w_i = \sum_{\substack{j \text{ ASSIGNED} \\ \text{TO LINK } i}} t_j$$



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COST FOR AGENT i (ONE AGENT PER LINK)

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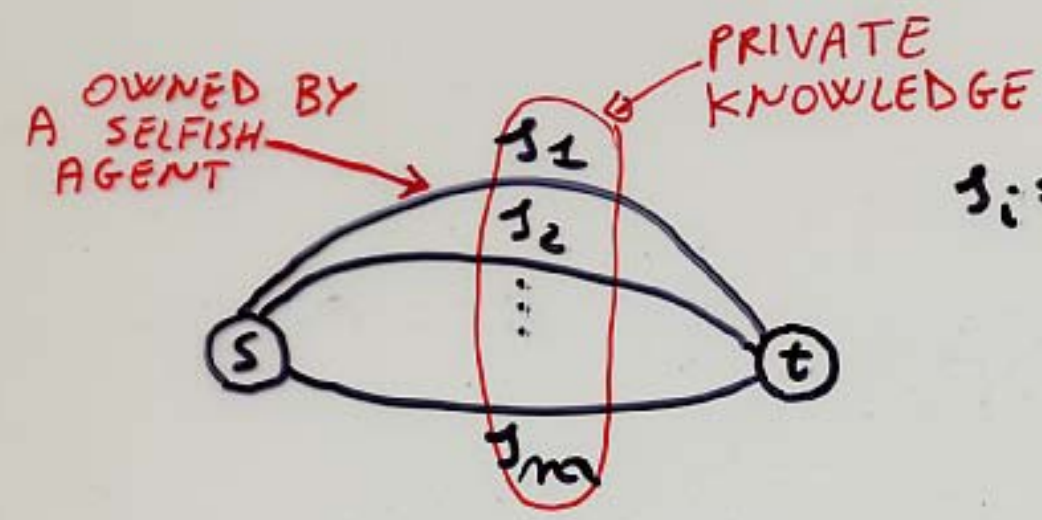

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AGENTS MAY LIE (REPORT $r_i \neq s_i$)

IDEA: PAY THE AGENTS TO INDUCE TRUTHFUL BEHAVIOR ($r_i = s_i$)

MECHANISMS

$$M = (ALG, P)$$

$$ALG(r_1, \dots, r_i, \dots, r_m) \qquad P_i(r_1, \dots, r_i, \dots, r_m)$$

$$\Downarrow$$

$$COST_i = \frac{w_i^{ALG}(r_1, \dots, r_i, \dots, r_m)}{s_i}$$

UTILITY (NET PROFIT):

$$u_i(r_1, \dots, r_i, \dots, r_m) = P_i(\dots r_i \dots) - COST_i$$

CAN WE USE EVERY ALG?

NO!!

- $M = (ALG, P)$ TRUTHFUL $\Rightarrow$

  ALG MUST BE MONOTONE

- ALG MONOTONE $\Rightarrow \exists\ P_{ALG}$ s.t.

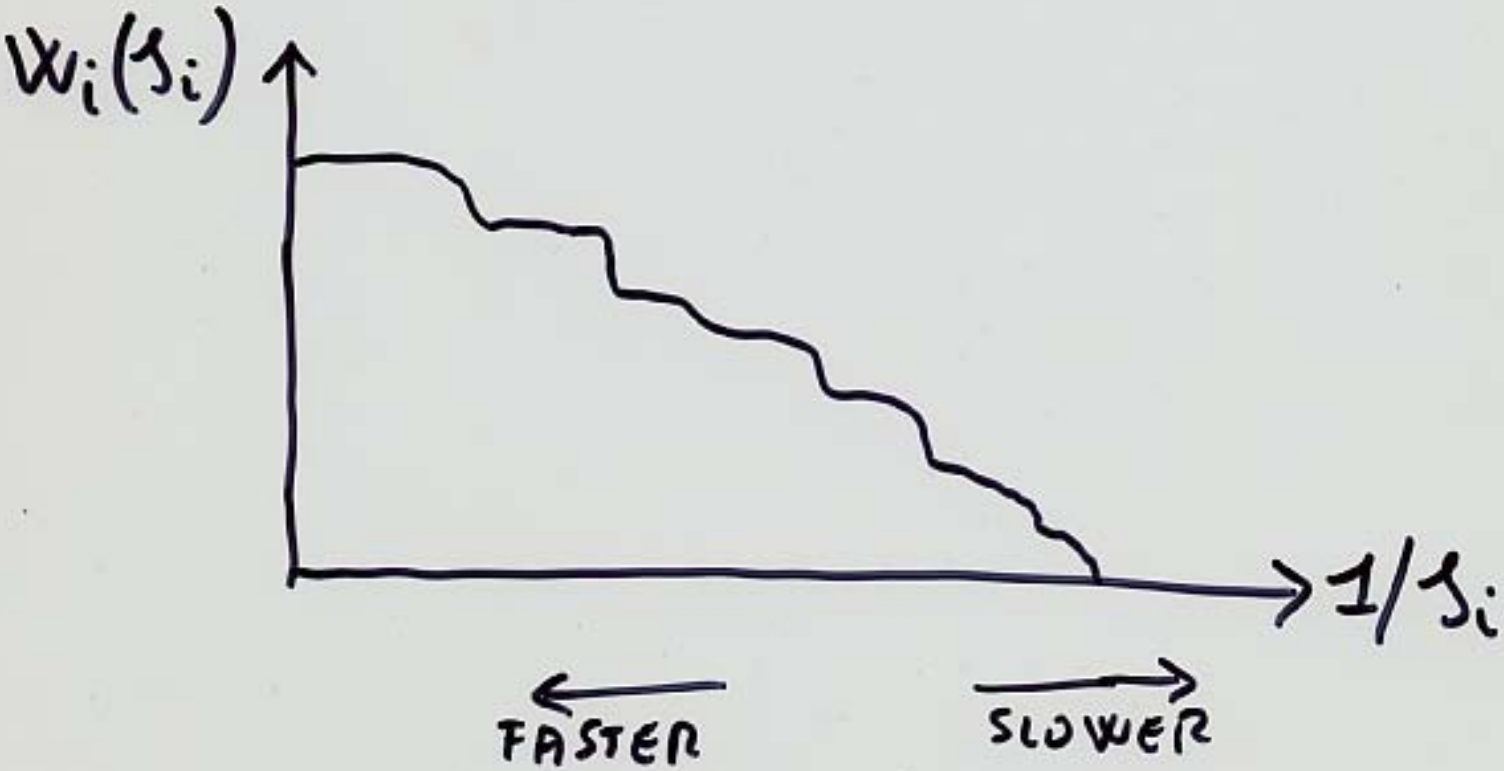
  (ALG, P_{ALG}) IS TRUTHFUL

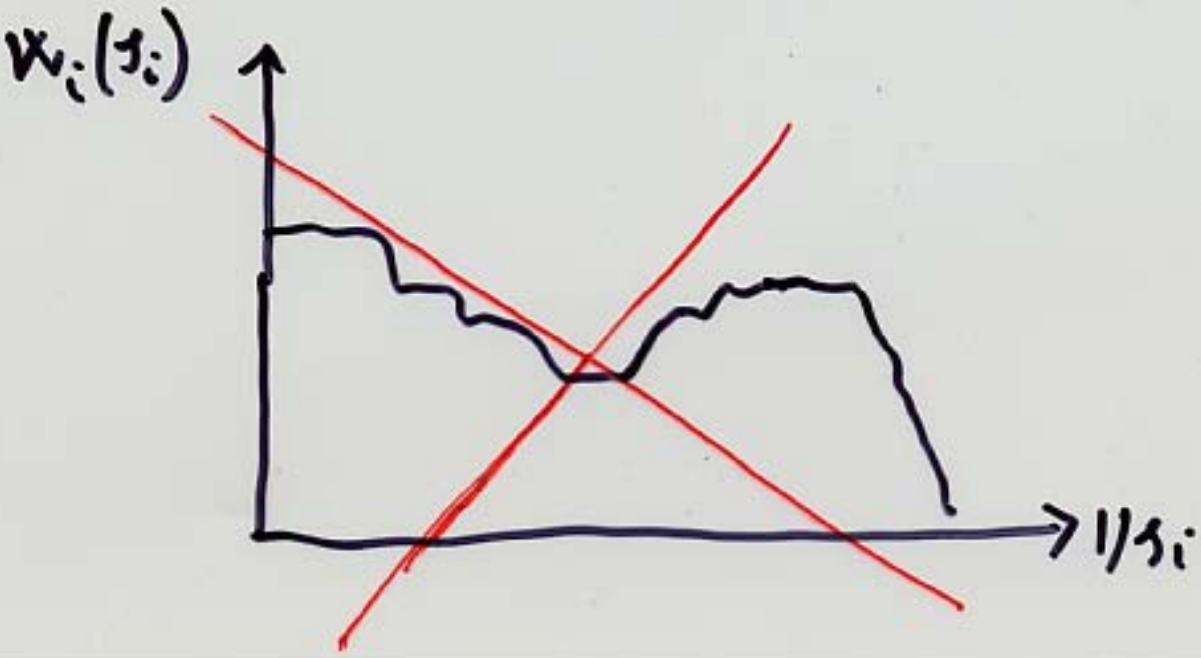
[ARCHER-TARDOS, 2001]

MONOTONE ALGOS

FIX $s_{-i} = s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_m$

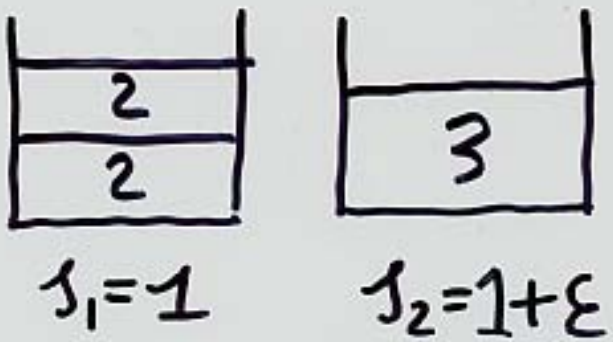
$$W_i(s_i) = w_i^{ALG}(s_1, \dots, s_{i-1}, s_i, s_{i+1}, \dots, s_m)$$

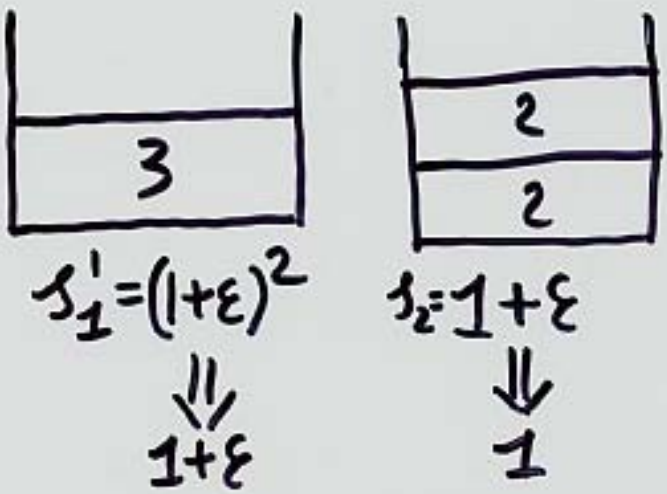


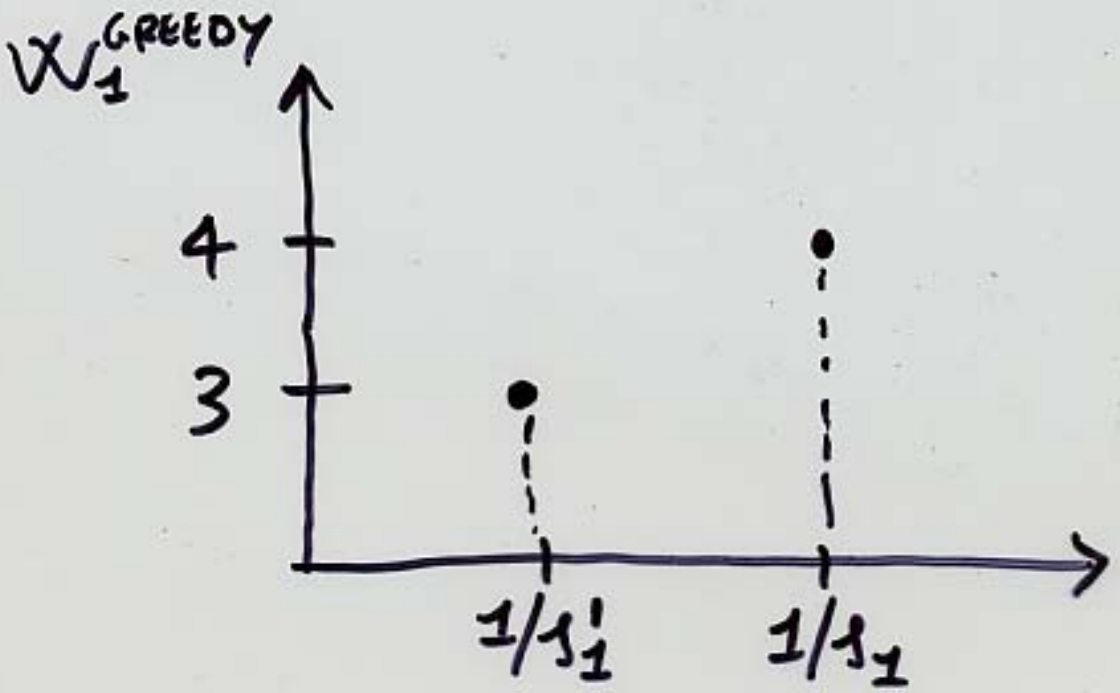


GREEDY IS NOT MONOTONE

JOBS: 3, 2, 2



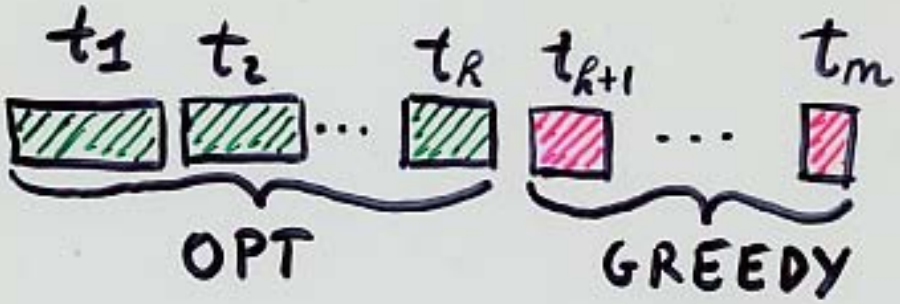




PROBLEM VERSION	MONOTONE APX ALGORITHMS		
ANY m	1	EXP TIME	[ARCHER-TARDOS OI
	$3+\varepsilon$	POLYTIME RANDOMIZED TRUTHFUL IN EXPECTATION	
$m \in O(1)$			
$s_{MAX} \in O(1)$	$2+\varepsilon$	DETERMINISTIC POLYTIME	
DIVISIBLE SPEEDS $(s_i = 2^{c_i})$	$2+\varepsilon$	"	[THIS WORK]
ANY SPEEDS	$4+\varepsilon$	"	

PROBLEM VERSION	MONOTONE APX ALGORITHMS	
ANY m	1	EXP TIME [ARCHER-TARDOS 01]
	$3+\varepsilon$	POLYTIME RANDOMIZED TRUTHFUL IN EXPECTATION ← WEAKER NOTION
$m \in O(1)$	$2+\varepsilon$	DETERMINISTIC POLYTIME
$s_{max} \in O(1)$		
DIVISIBLE SPEEDS $(s_i = 2^{c_i})$	$2+\varepsilon$	" [THIS WORK]
ANY SPEEDS	$4+\varepsilon$	" ← DOMINANT STRATEGIES

OVERVIEW



PTAS

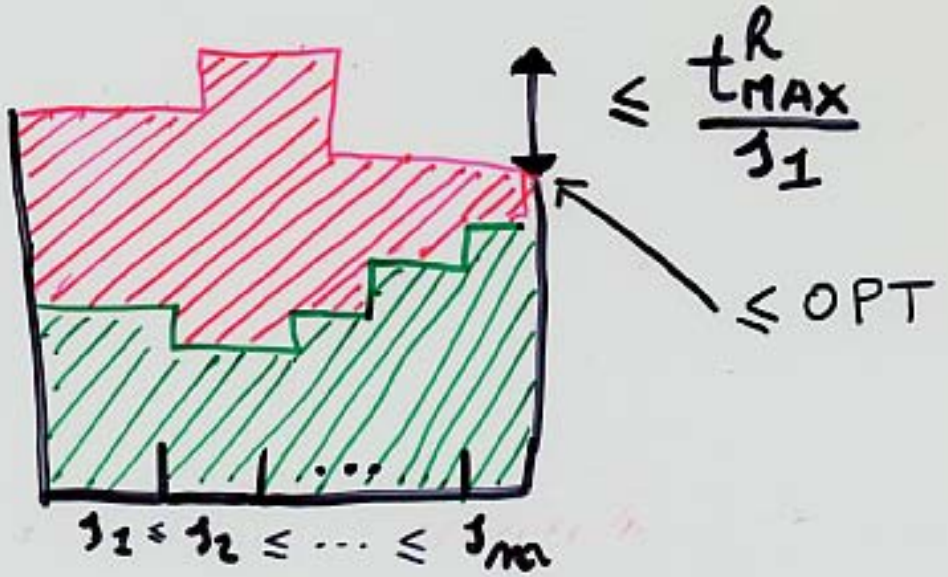
$(1+\varepsilon)$-APX

PTAS-INDEPENDENT

$(2+\varepsilon)$-APX

PTAS-MONOTONE:

REPLACE GREEDY WITH A MONONONE "SUFFICIENTLY GOOD" ALGORITHM



$$t^R_{MAX} = MAX\{t_{R+1}, \ldots, t_m\} \leq t_R$$

$$APX \leq OPT + \frac{t_R}{s_1} \qquad OPT \geq \frac{R \cdot t_R}{\sum s_i}$$

$$APX \leq OPT + \frac{OPT}{R \cdot s_1} \cdot \sum s_i$$

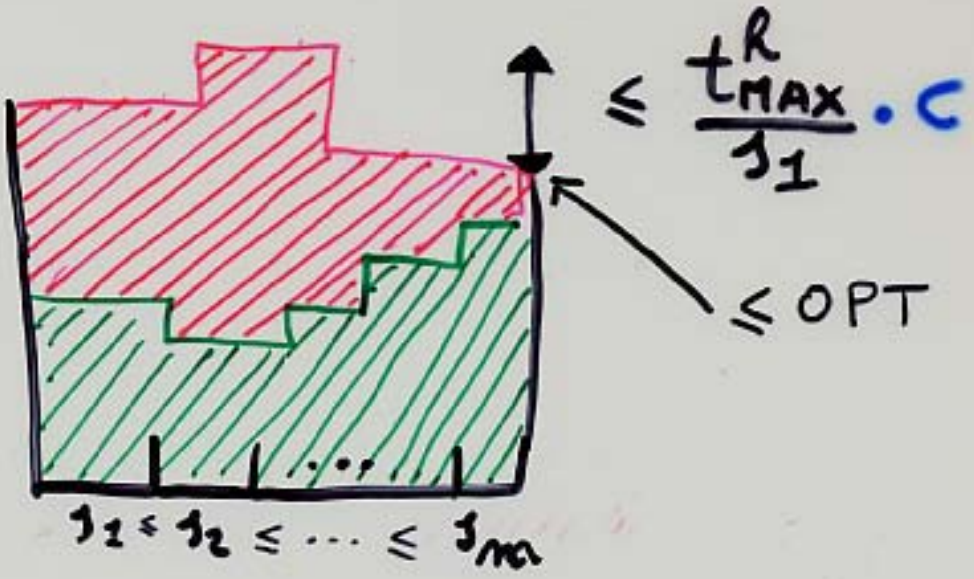
$$= OPT\left(1 + \frac{\sum s_i}{R \cdot s_1}\right)$$

$$\leq OPT\left(1 + \frac{m \cdot s_m}{s_1}\right)$$

$s_m \gg s_1$ ⇓ IGNORE MACHINE s_1

$s_m \approx s_1$ ⇓ $m/R = \varepsilon$

C-GREEDY-CLOSE ALGO



$$t^R_{MAX} = MAX\{t_{R+1}, \ldots, t_m\} \leq t_R$$

$$APX \leq OPT + \frac{t_R}{s_1} \cdot c \qquad OPT \geq \frac{R \cdot t_R}{\sum s_i}$$

$$APX \leq OPT + \left(\frac{OPT}{R \cdot s_1} \cdot \sum s_i\right) \cdot c$$

$$= OPT \left(1 + \frac{\sum s_i}{R \cdot s_1}\right) \times c$$

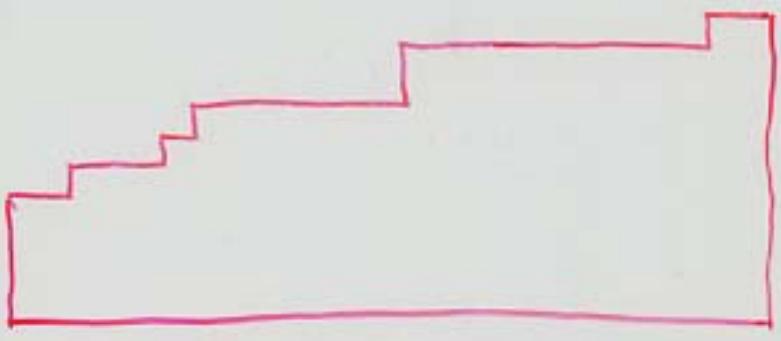
$$\leq OPT \left(1 + \frac{m \cdot s_m}{s_1}\right) \times c$$

$s_m \gg s_1$ ⇓ IGNORE MACHINE s_1

$s_m \approx s_1$ ⇓ $c \cdot m/R = \varepsilon$

1) RUN GREEDY ON $S = \Sigma_1$: IDENTICAL MACHINES

2) ORDER THE LOADS



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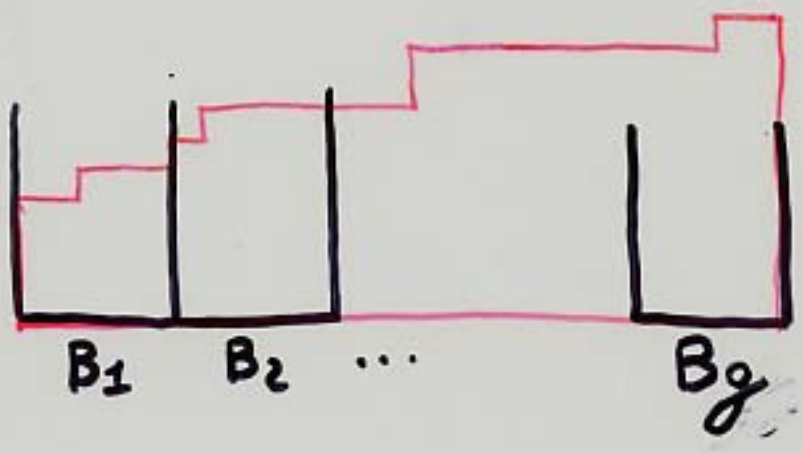

3) CONSIDER $g=GCD(\ell_1,\dots,\ell_m)$ BLOCKS

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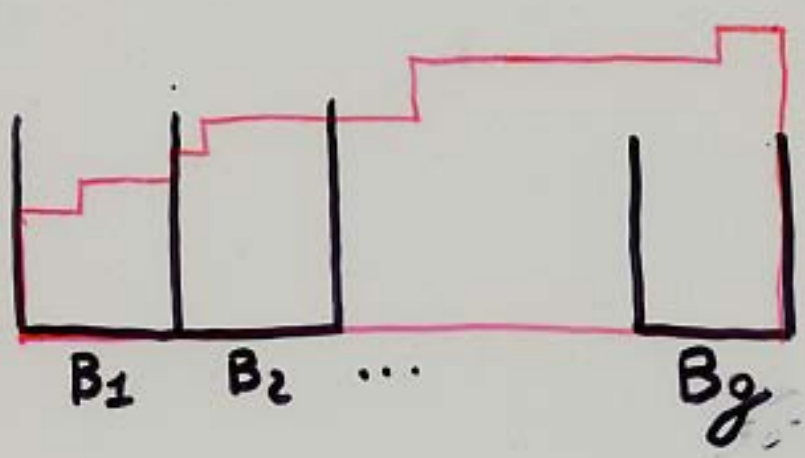

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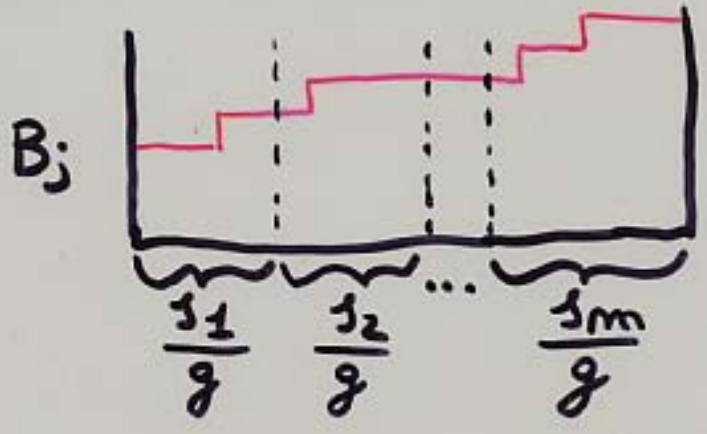
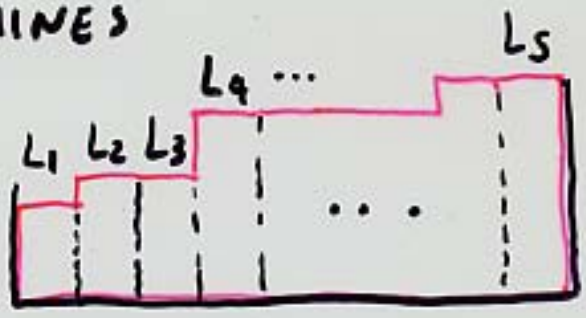
4) ASSIGN "BLOCKWISE"



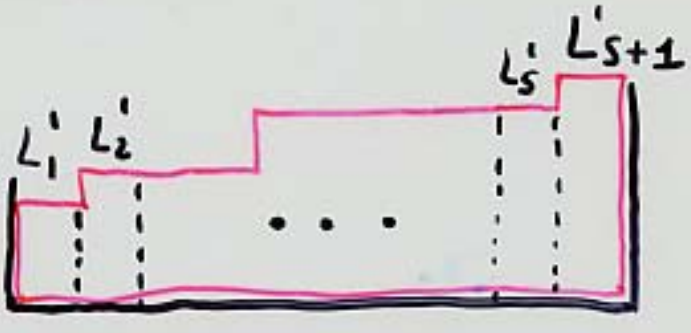
THEOREM: $COST \leq OPT + \frac{t_{MAX}}{g}$

MONOTONICITY

S MACHINES



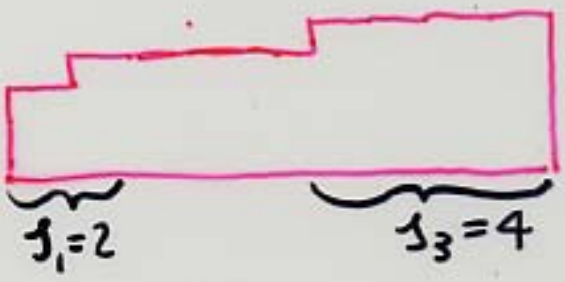
S+1 MACHINES



$L'_i \leq L_i$ $\qquad$ $L'_{S+1-i} \leq L_{S-i}$

$J_2 = 2$

$J'_2 = 3$



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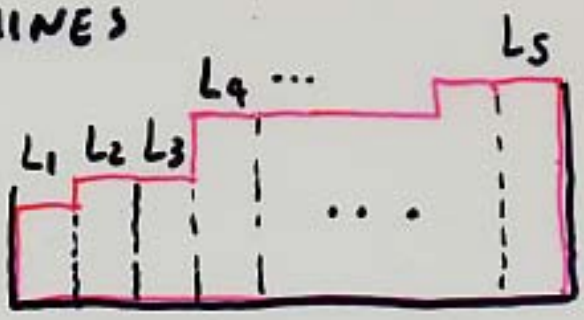

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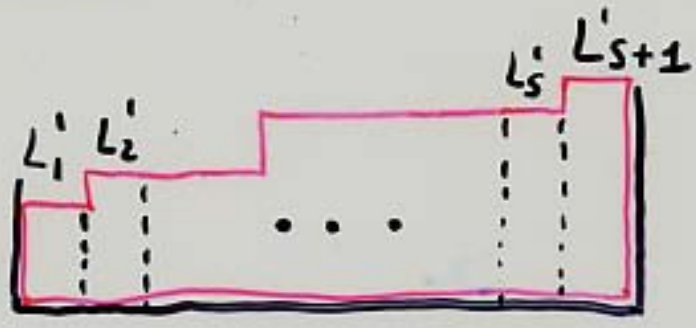

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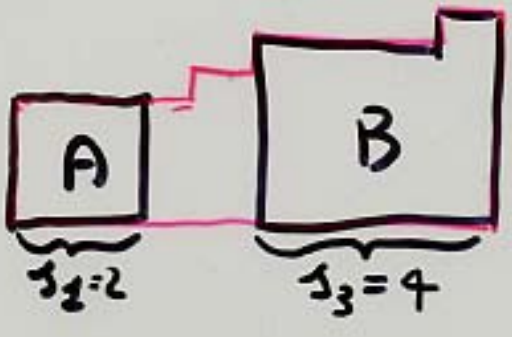

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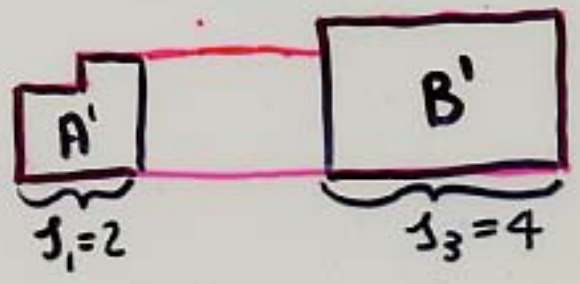

CONCLUSIONS

- FIRST POLYTIME DETERMINISTIC $O(1)$-APX MECHANISM
  (TRUTHFUL IN STRONGEST MODEL)
- PAYMENTS P_i POLYTIME COMPUTABLE

OPEN QUESTIONS

1) $O(1)$-APX DETERMINISTIC FOR $m \notin O(1)$
2) $(1+\varepsilon)$-APX
   - $m \in O(1)$, $s_{MAX} \in O(1)$
   - DIVISIBLE SPEEDS