Imperfect Best-Response Mechanisms

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joint work with
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Best-response mechanisms [Nisan et al., 2011]

- At each time step, a subset of agents is adversarially chosen
- The selected agents adopt their best-response
- Repeat until the equilibrium has been reached
- Agents utilities/costs are only evaluated at the equilibrium

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Examples

- BGP
- some TCP variants
- GSP auctions
- Interns-Hospital Matching (IHM)

Convergence & Incentive-Compatibility

Convergence

- The dynamics will eventually converges to a Nash equilibrium

Convergence & Incentive-Compatibility

Convergence

- The dynamics will eventually converges to a Nash equilibrium

Incentive Compatibility

- If a player does not play the best response whenever is selected, the dynamics will reach a different equilibrium
- The utility for this player at new equilibrium is lower than in the equilibrium reached by always playing the best response

NBR-solvable games [Nisan et al., 2011]

NBR-solvable game

- NBR strategy: a strategy that can never be a best-response
- A game solvable by iterated elimination of NBR strategies

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Clear outcome

- A NBR solvable game has clear outcome if for each player i...
- ...there is a sequence of eliminations of NBR strategies...
- ...such that the equilibrium maximizes the utility of i...
- ...at the first time that i eliminate a strategy in this sequence

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BGP, TCP, GSP & IHM are NBR-solvable with clear outcomes

In this work...

Theorem (Nisan et al., 2011)

- *If a game is NBR-solvable, then the best-response mechanism converges*
- *If the NBR-solvable game has a clear outcome, then the best-response mechanism is also incentive-compatible*

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Our contribution

- What happen if an agent can sometimes take a wrong action?
- How resistant are these results to small perturbations?
- Are convergence and incentive-compatibility robust?

Imperfect best-response mechanisms

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Imperfect best-response mechanisms

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p-imperfect best-response mechanism

- At each time step, a subset of agents is chosen by a non-adaptive adversary
- The selected agents adopt their best-response, except with probability p
- Repeat until the equilibrium has been reached
- Agents utilities/costs are only evaluated at the equilibrium

Does the convergence result holds?

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Obviously, if p is small...

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Obviously, if p is small...

WRONG!

- Even for p exponentially small in the number of players...
- there is a schedule of players such that for any $t > 0$...
- the p-imperfect mechanism is in the equilibrium at time t...
- with probability at most ε

Convergence: a negative result

The game

- n players with strategies s_0 and s_1
- player i prefers strategy s_1 only if $1, \dots, i-1$ are playing s_1

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- if $1, \ldots, i-1$ play s_1, player i gets wrong with probability p
- otherwise, she gets the wrong strategy with probability $q \ll p$

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  - Between two consecutive occurrence of i always appears $j > i$
  - The length of the sequence is 2^{n-1}
  - n appears only at the end of the sequence
- if $p = \Omega\left(\frac{1}{2^{n-1}}\right)$ and $q \to 0$, then n always plays s_0 w.h.p.

Convergence: a positive result

Convergence is not robust

- For best-response mechanisms, convergence result holds regardless of the schedule
- For p-imperfect mechanism, convergence results must depend on the schedule

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- then a p-imperfect mechanism converges...

Convergence: a positive result

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A positive result

- If p is small enough and the game is NBR-solvable...
- then a p-imperfect mechanism converges...
- but the bound on p depends on the schedule

Incentive-compatibility: a negative result

	left	right
top	$2, 1$	$1, 0$
bottom	$0, 0$	$0, c$

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- If the row player gets wrong with prob. p and $c = \Omega(1/p)$, then the column player prefers to play `right`

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We need a quantitative definition of clear outcome

Incentive-compatibility: a positive result

Theorem

A p-imperfect mechanism is incentive-compatible if for each i

$$u_i(NE) \geq \frac{1}{1-2\delta}\left(2\delta \cdot u_i^\star + u_i^k\right)$$

- $\delta = \delta(p) > 0$
- u_i^k*: max utility player i achieves at her first elimination*
- $u_i^\star$*: max utility player i achieves in the entire game*

Proof idea.

- If the player follows the p-imperfect mechanism...
- ...then she gets $u_i(NE)$
- Otherwise she gets at most $u_i^\star$ with prob. depending on p...
- ...and she gets at most u_i^k with remaining probability □

What happens for larger classes of games?

Different behavior for different schedules

	0	1
0	1,1	0,0
1	0,0	1,1

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Different behavior for different schedules

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0	1,1	0,0
1	0,0	1,1

Different behavior for different best-response mechanisms

	0	1
0	0,0	0,1
1	0,1	1,0

Other results

- We try to describe how p-imperfect mechanism behave

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- … with an application to PageRank games

Thank you!