ONLINE LOAD BALANCING MADE SIMPLE: GREEDY STRIKES BACK

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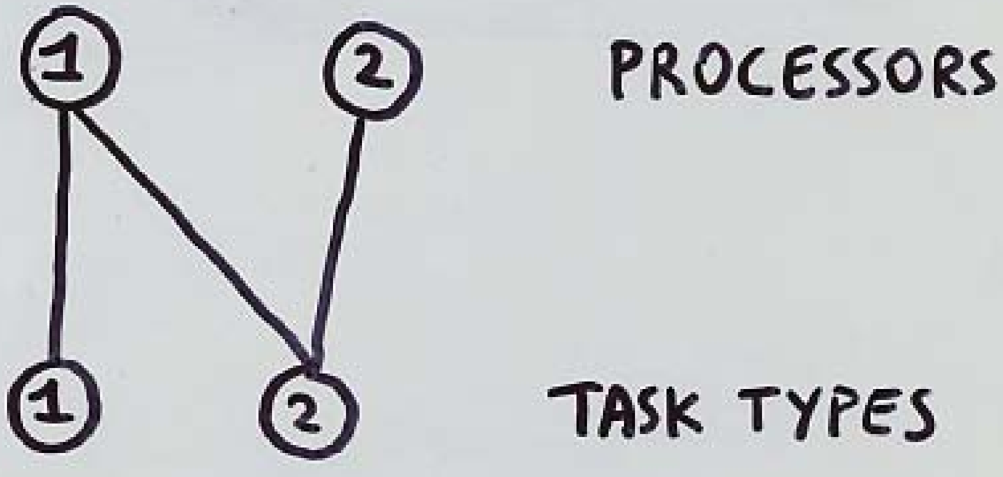
5 RWTH AACHEN

PROBLEM DEFINITION

ONLINE LOAD BALANCING VERSION:

- WEIGHTED TEMPORARY TASKS
- UNKNOWN DURATION
- RESTRICTED ASSIGNMENT
- NO PREEMPTION

<u>GOAL</u>: MINIMIZE MAX LOAD



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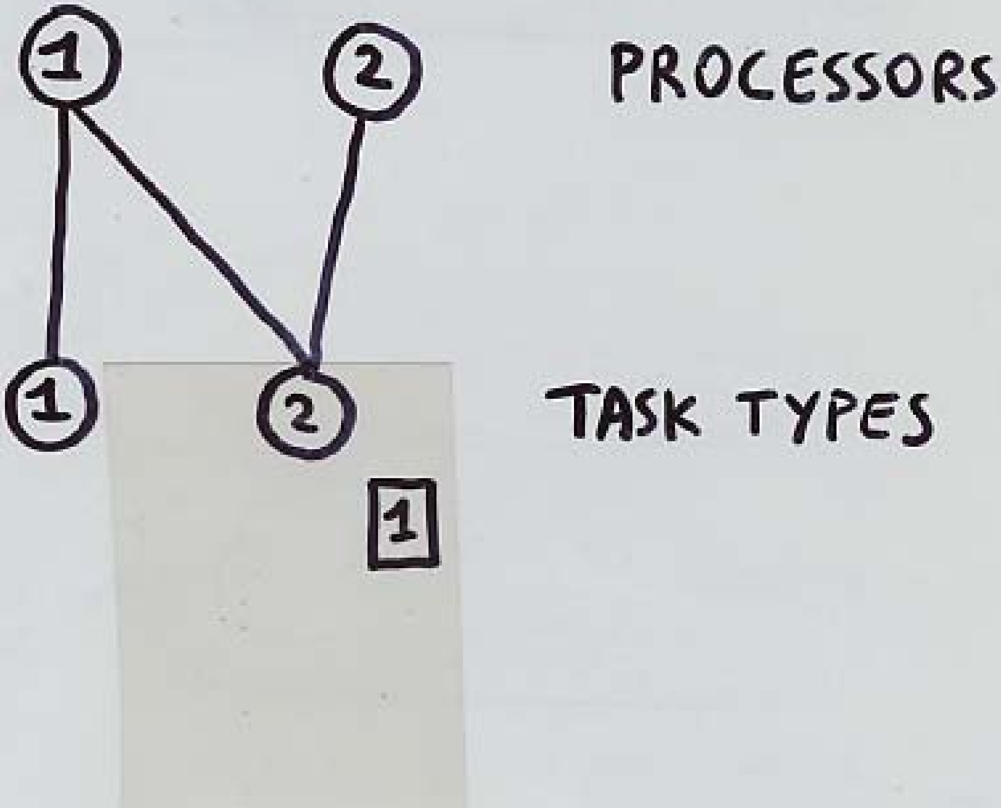

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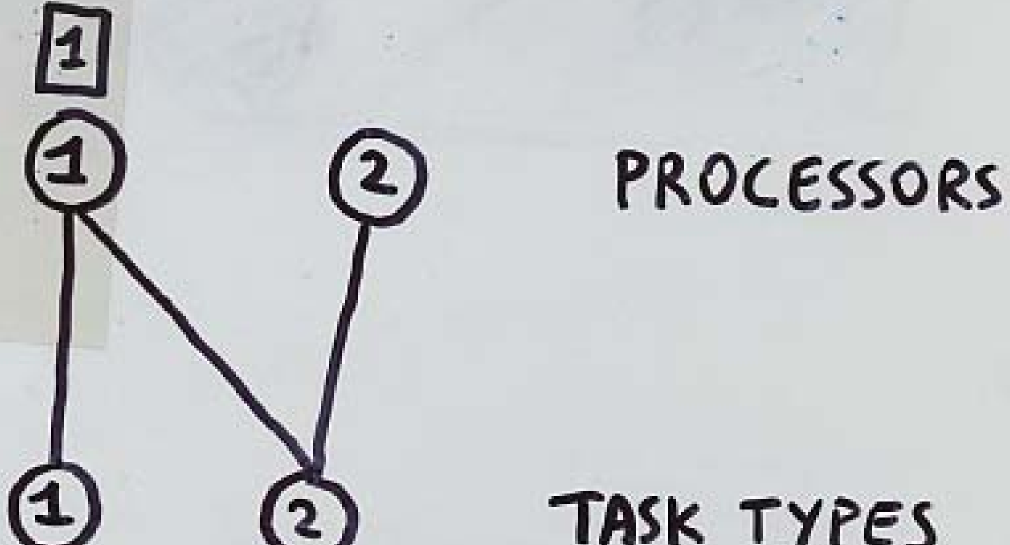

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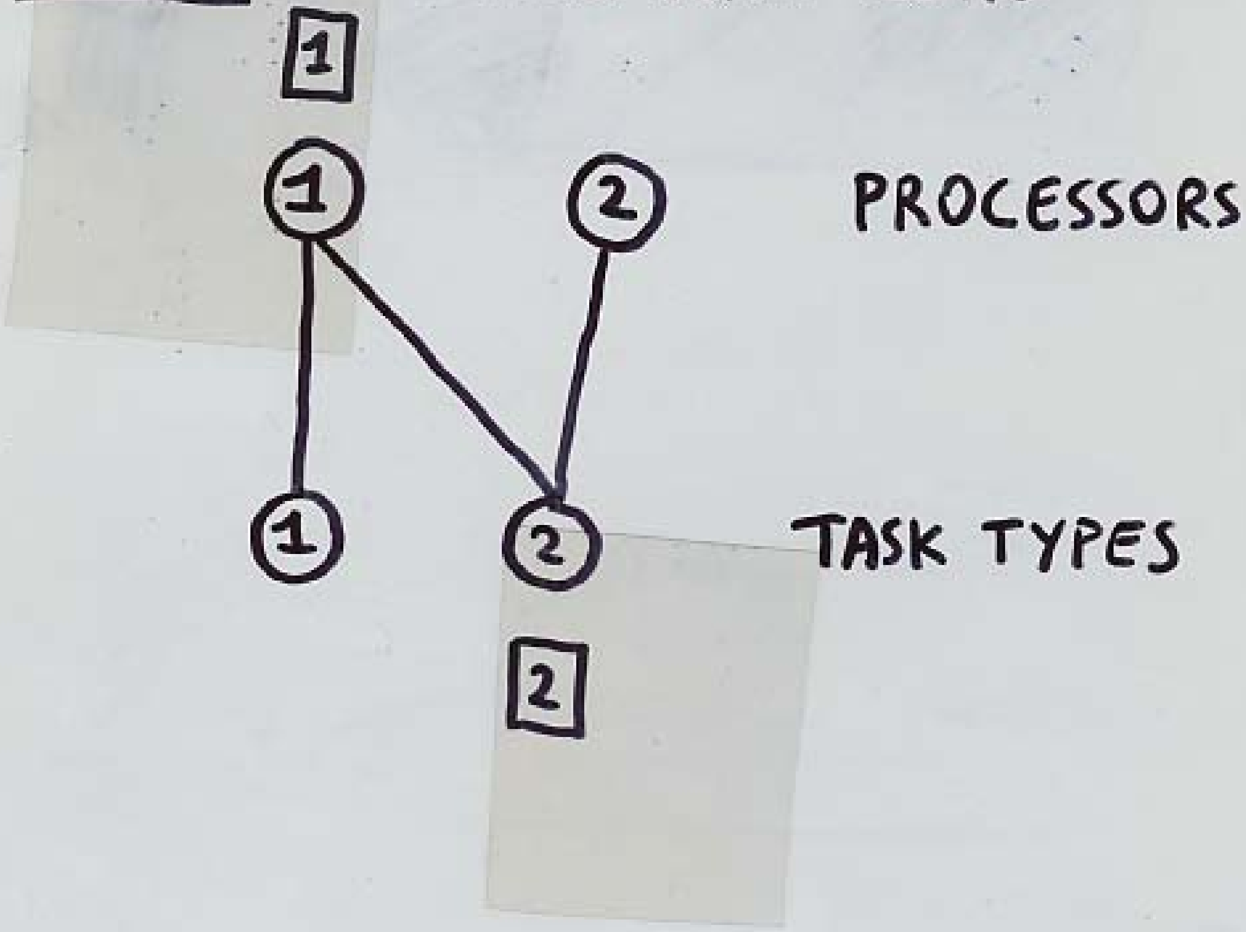

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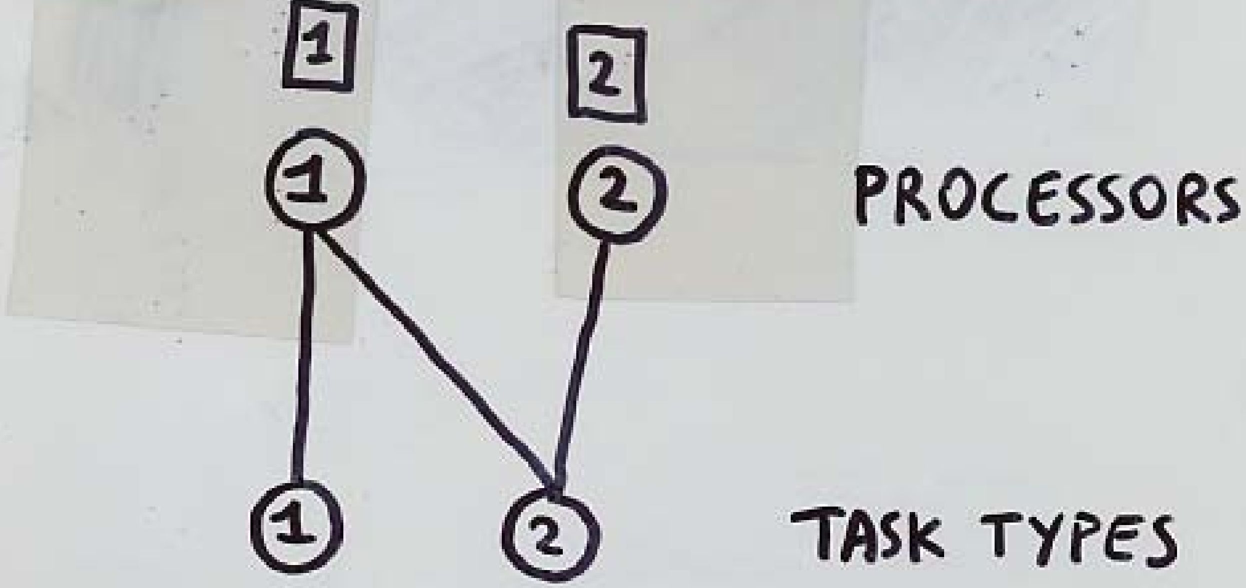

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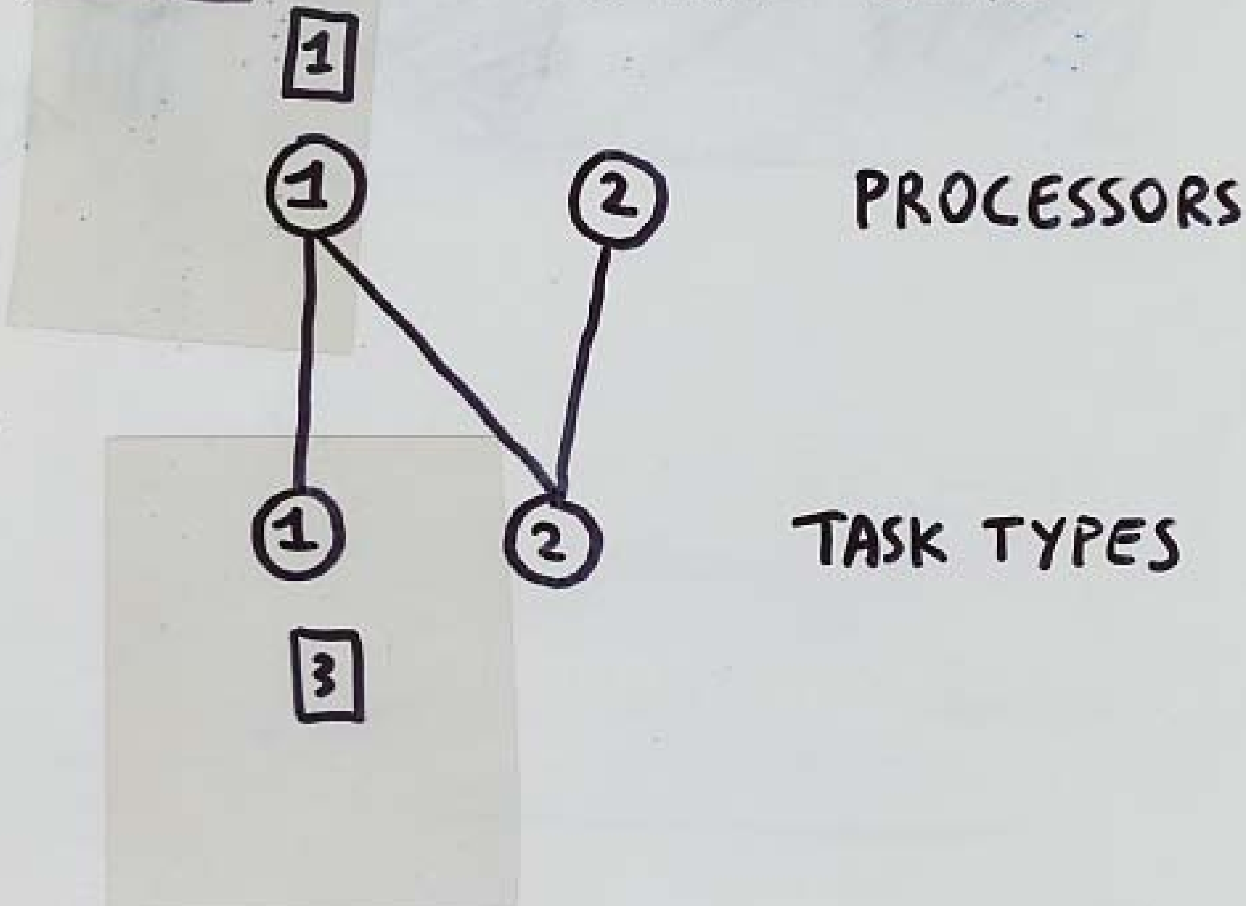

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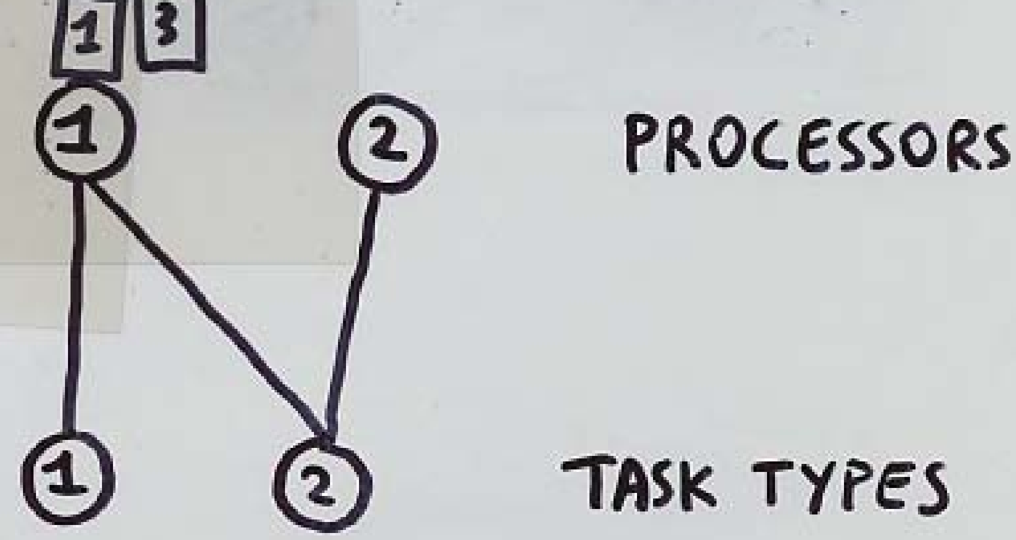

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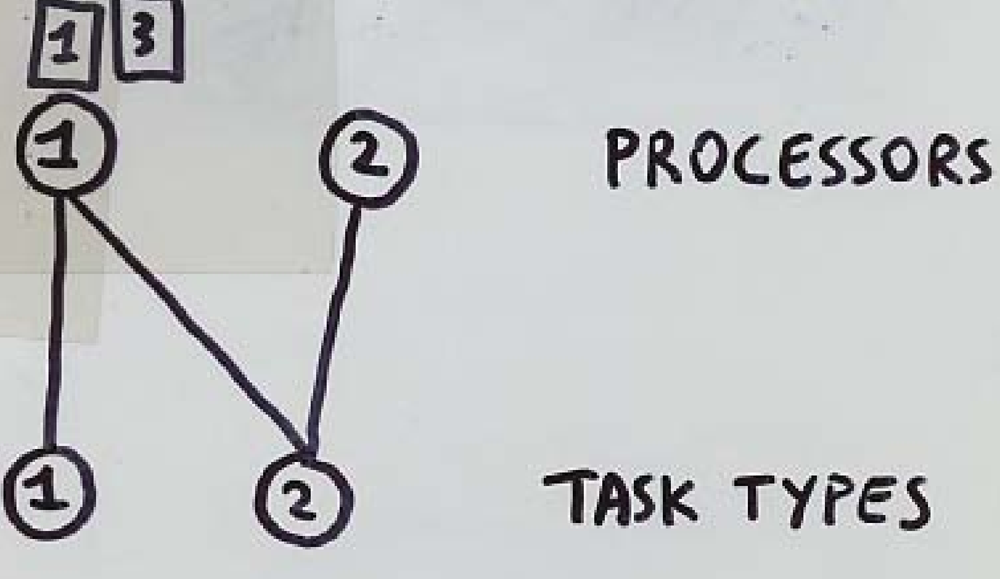

ALG IS C-COMPETITIVE IF

$$\exists b : \forall \sigma \quad \mathrm{COST}(\mathrm{ALG}(\sigma)) \leq c \cdot \mathrm{OPT}(\sigma) + b$$

PREVIOUS WORK

GENERAL CASE (ANY GRAPH)

- GREEDY IS $\Omega(m^{2/3})$-COMPETITIVE [AZAR ET AL '95]
- OPTIMAL $\Theta(m^{1/2})$-COMPETITIVE ALG. [AZAR ET AL '95,97]

UNRESTRICTED CASE (GRAPH $K_{1,m}$)

- GREEDY IS $(2-1/m)$-COMPETITIVE [GRAHAM '66]
- GREEDY IS OPTIMAL [AZAR & EPSTEIN '97]

HIERARCHICAL TOPOLOGIES (SPECIAL INTERVAL GRAPHS)

- GREEDY IS $\Omega(\log m)$-COMPETITIVE [FOLKLORE]
- 5-COMPETITIVE ALGORITHM [BAR-NOY ET AL '99]

GREEDY ALGORITHM

SIMPLE AND DISTRIBUTED: QUERIES ONLY NEIGHBOR PROCESSORS

THE OPTIMAL ALGORITHMS FOR THE GENERAL AND THE HIERARCHICAL CASES

- ARE NOT "LOCAL"
- NEED TO ESTIMATE OPT(α)
- NON TRIVIAL ANALYSIS

QUESTIONS

DOES GREEDY "FAIL" BECAUSE IT ONLY USES LOCAL INFORMATION?

ARE THERE OPTIMAL (DISTRIBUTED) ALGORITHMS WITH THE NICE FEATURES OF THE GREEDY ONE?

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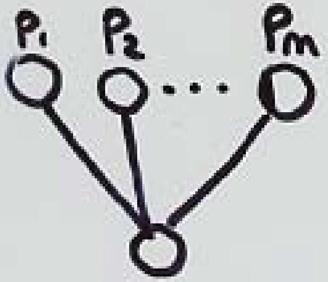
DOES GREEDY "FAIL" BECAUSE IT ONLY USES LOCAL INFORMATION? NO [THIS WORK]

ARE THERE OPTIMAL (DISTRIBUTED) ALGORITHMS WITH THE NICE FEATURES OF THE GREEDY ONE?
YES [THIS WORK]

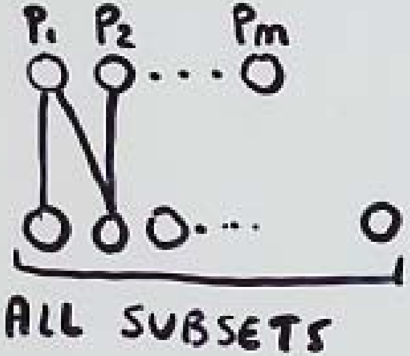
OBSERVE:

- "SIMPLE" GRAPHS → GREEDY PERFORMS WELL
  $O(1)$-COMPETITIVE



⟺ UNRESTRICTED CASE

- "HARD" GRAPH → GREEDY PERFORMS BADLY



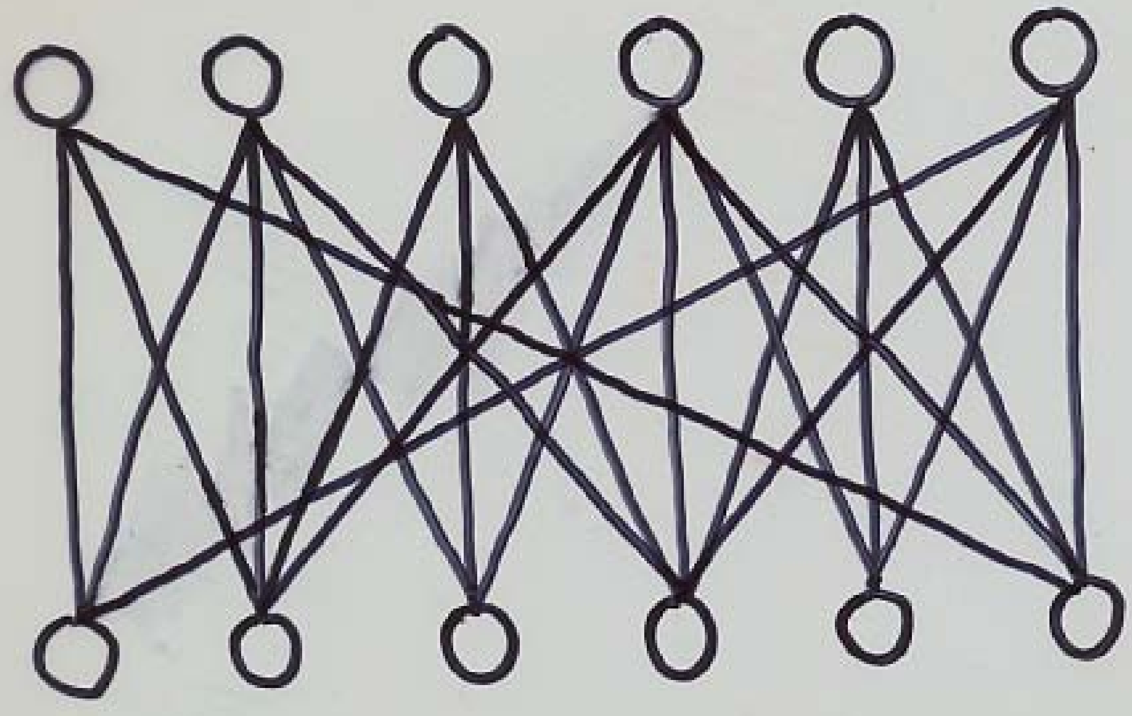
⟺ (GENERAL) RESTRICTED CASE

GREEDY $\Omega(m^{2/3})$

OPT ALG $\Theta(\sqrt{m})$

OUR APPROACH

IDEA: "MAKE G SIMPLE" AND USE GREEDY



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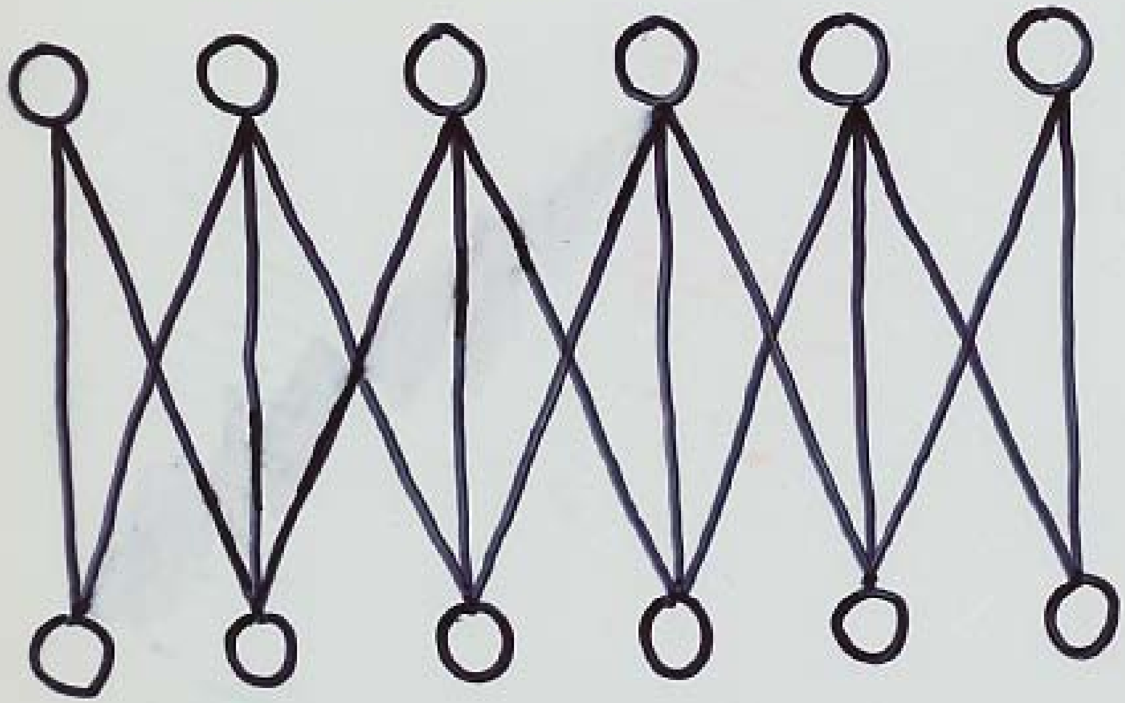

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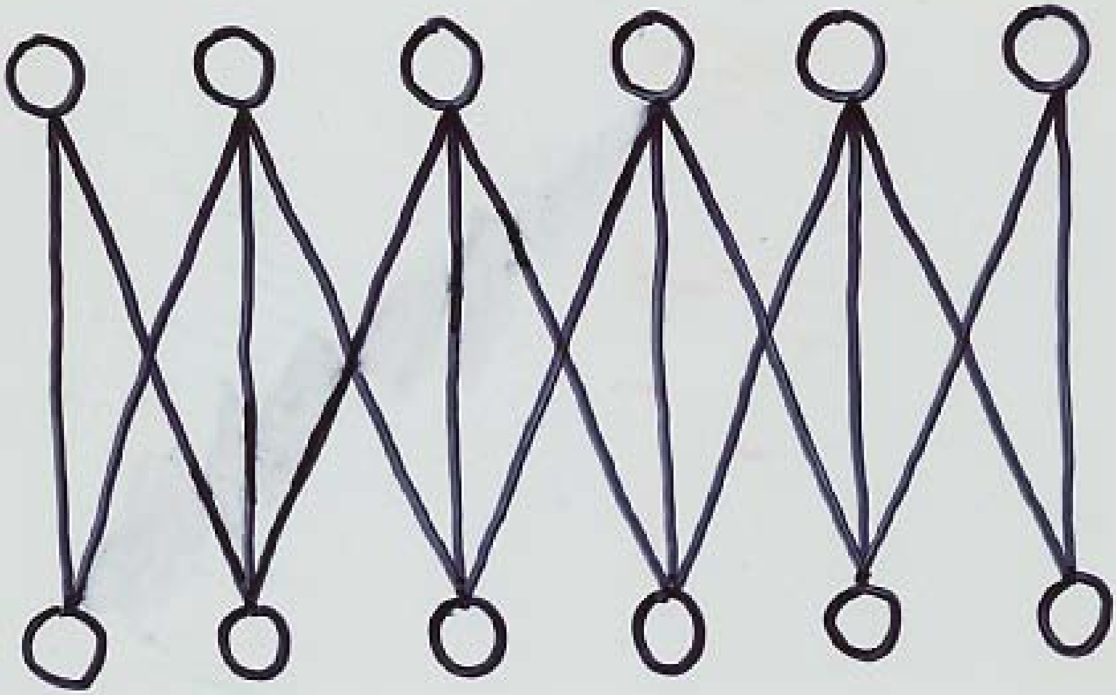

- PRECOMPUTE (OFFLINE)

  $G' = (V, E')$, $E' \subseteq E$

  $G' \subseteq G = (V, E)$
- RUN GREEDY ON G' (ONLINE)

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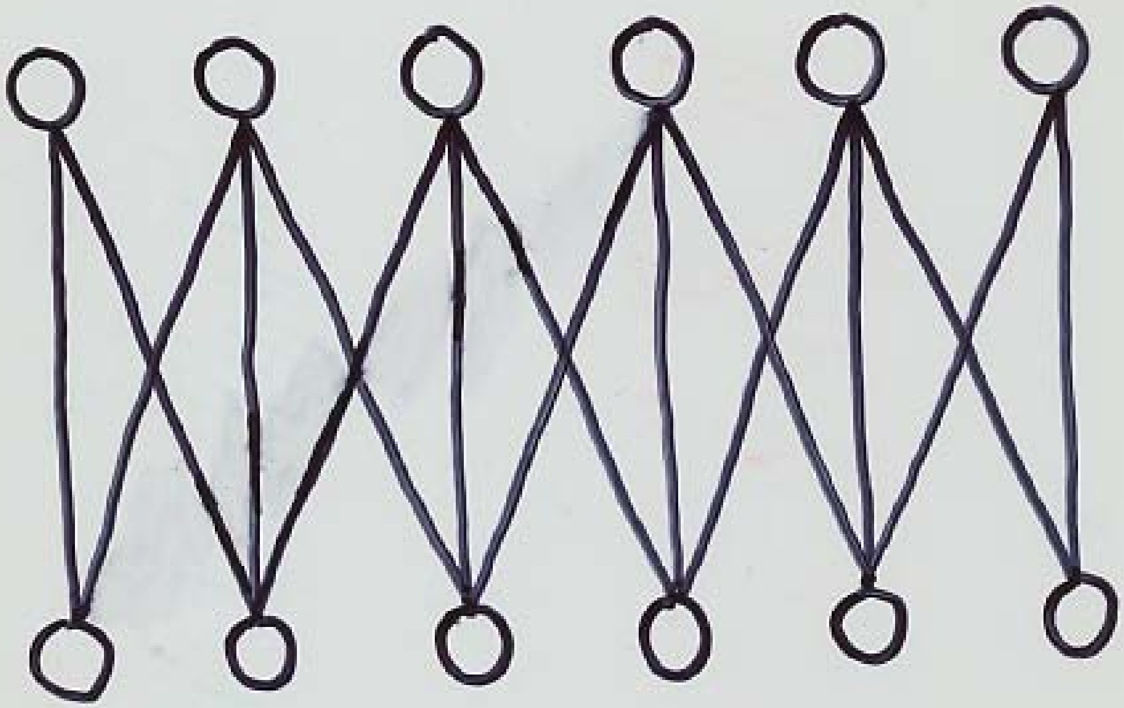

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$$G' = (V, E'), \quad E' \subseteq E$$

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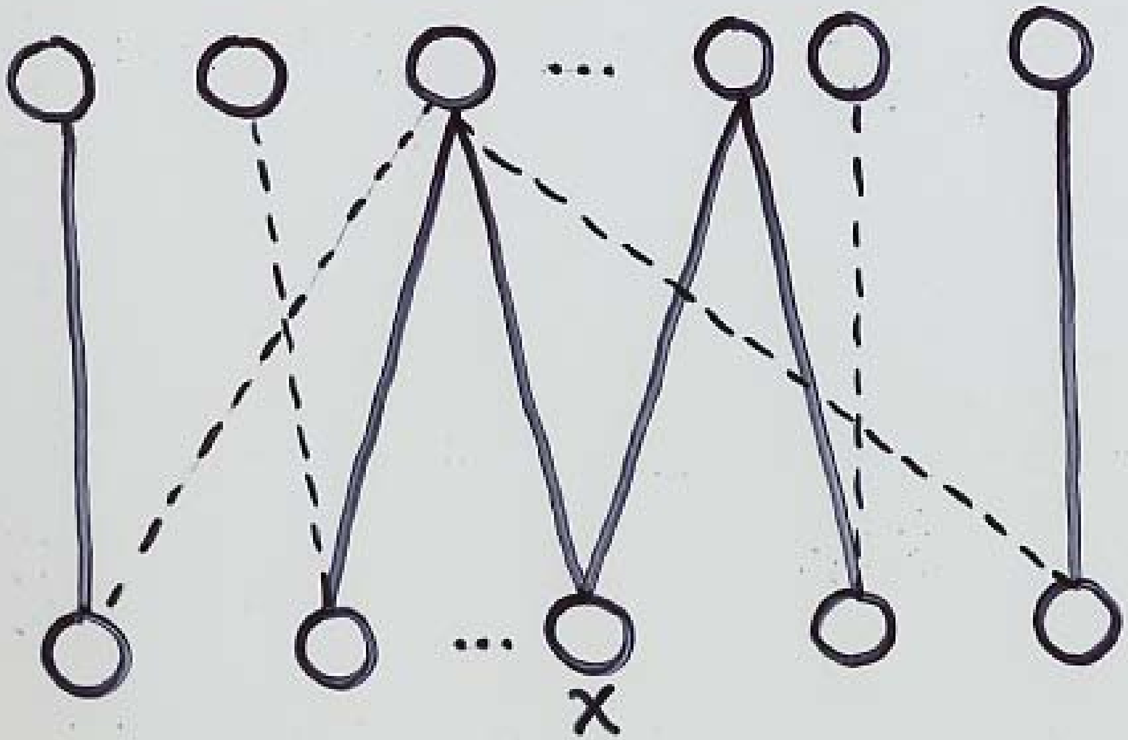
$$\underbrace{\boxed{\text{- RUN GREEDY ON } G' \text{ (ONLINE)}}}_{\text{SUB-GREEDY ALGORITHM}}$$

ANALYSIS



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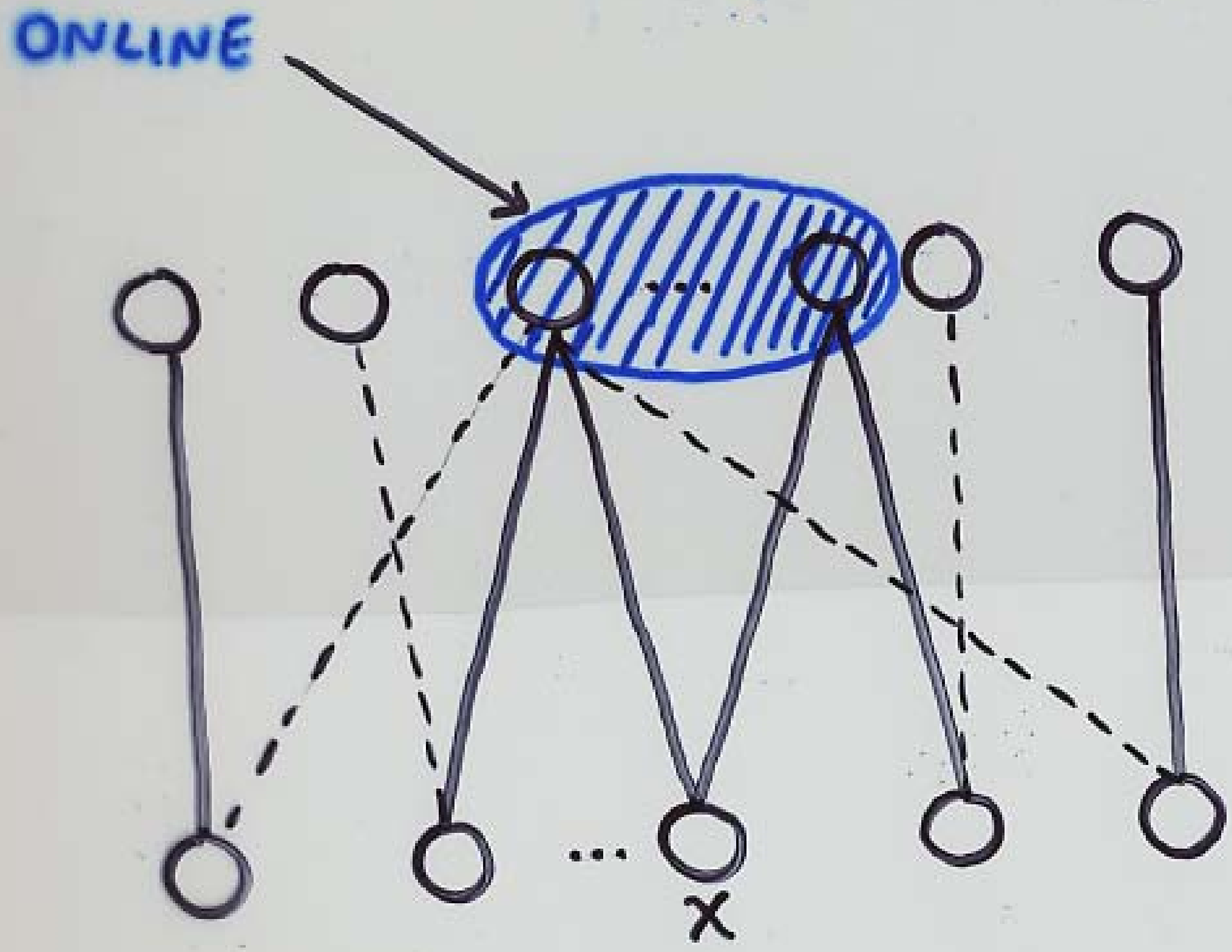

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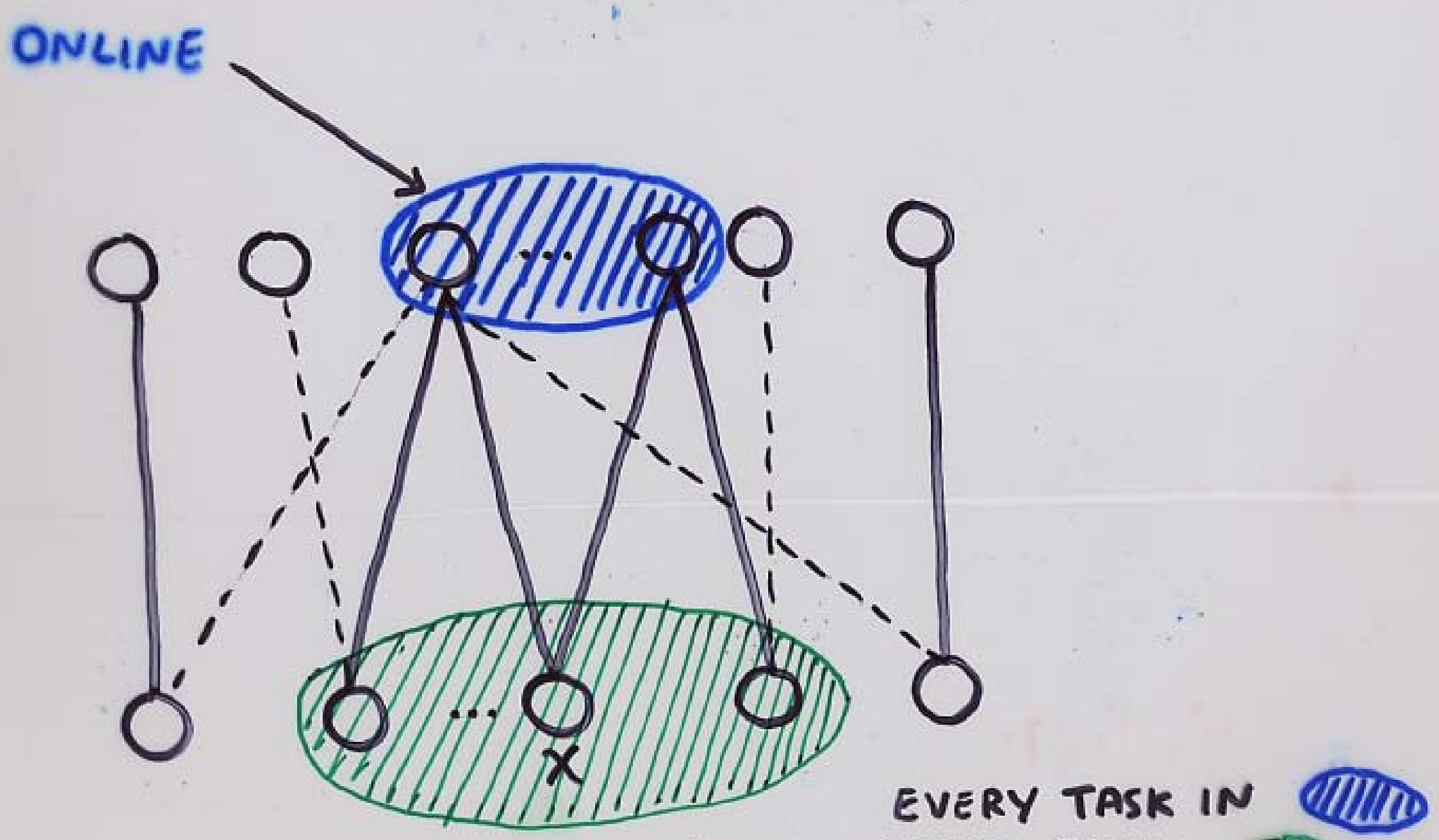

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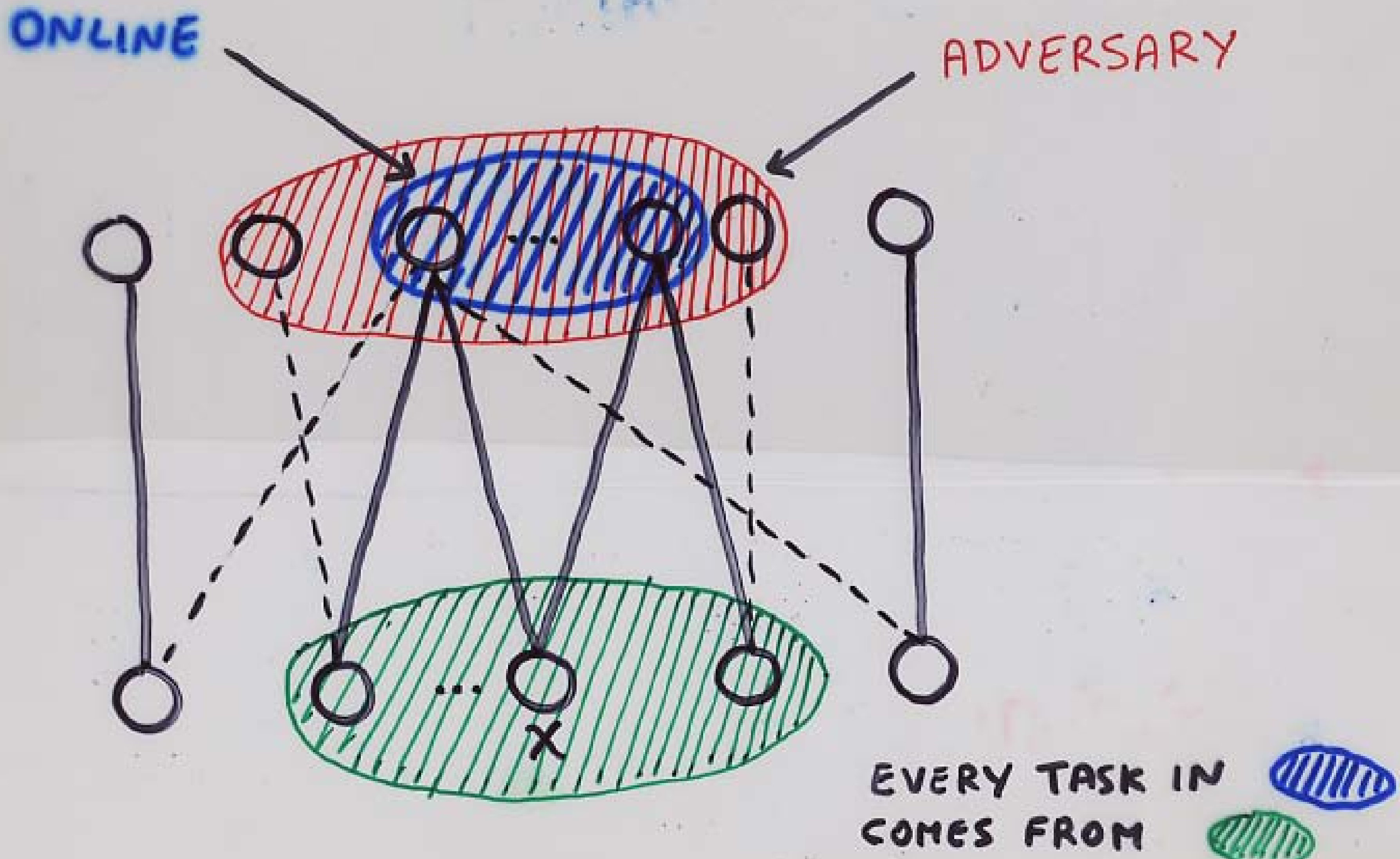


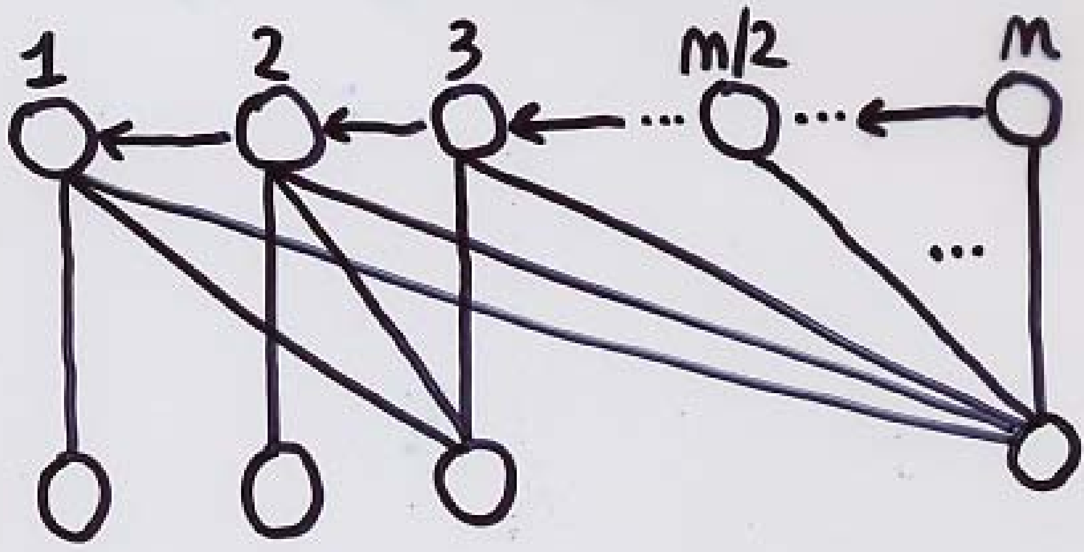
MAIN RESULT

$$\text{COMPETITIVE RATIO} \approx \frac{\text{ADVERSARY}}{\text{ONLINE}}$$

$$\text{WEIGHTED TASKS: } 1 + \max_{x \in \text{TASK TYPES}} \left\{ \frac{|\Gamma_G(\Gamma_{G'}(\Gamma_{G'}(x)))| - 1}{|\Gamma_{G'}(x)|} \right\}$$

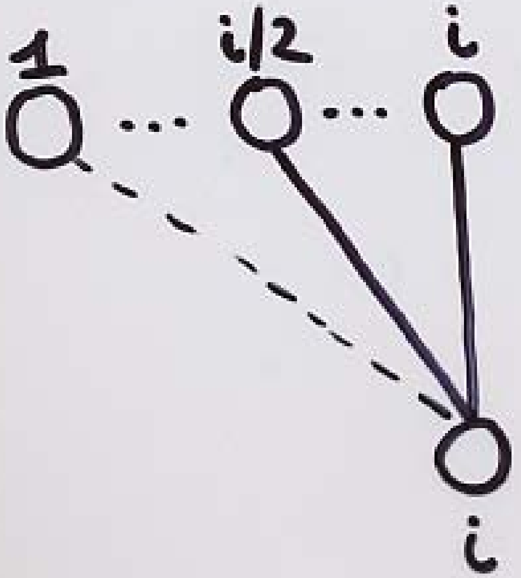
$$\text{UNWEIGHTED TASKS: } \max_{x \in \text{TASK TYPES}} \left\{ \frac{|\Gamma_G(\Gamma_{G'}(\Gamma_{G'}(x)))|}{|(\Gamma_{G'}(x))|} \right\}$$

HIERARCHICAL TOPOLOGIES



GREEDY: $\Omega(\log m)$-COMPETITIVE

SUB-GREEDY: 5-COMPETITIVE



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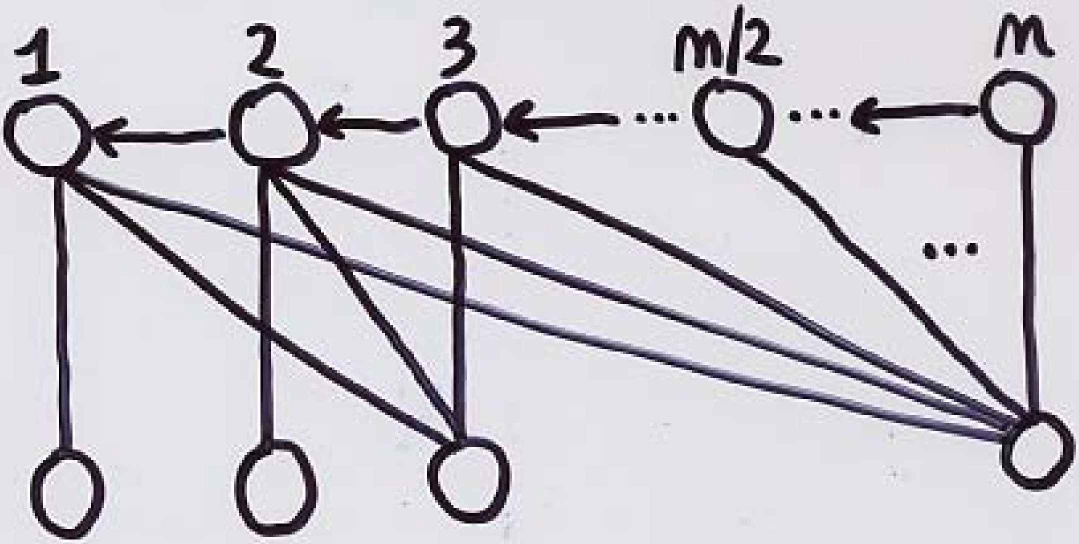

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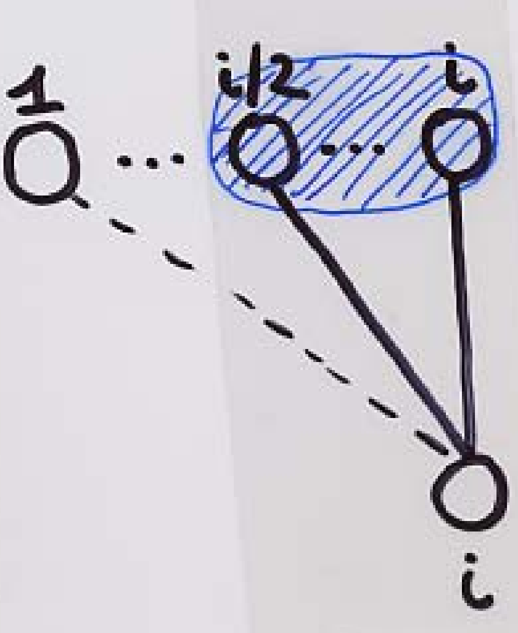

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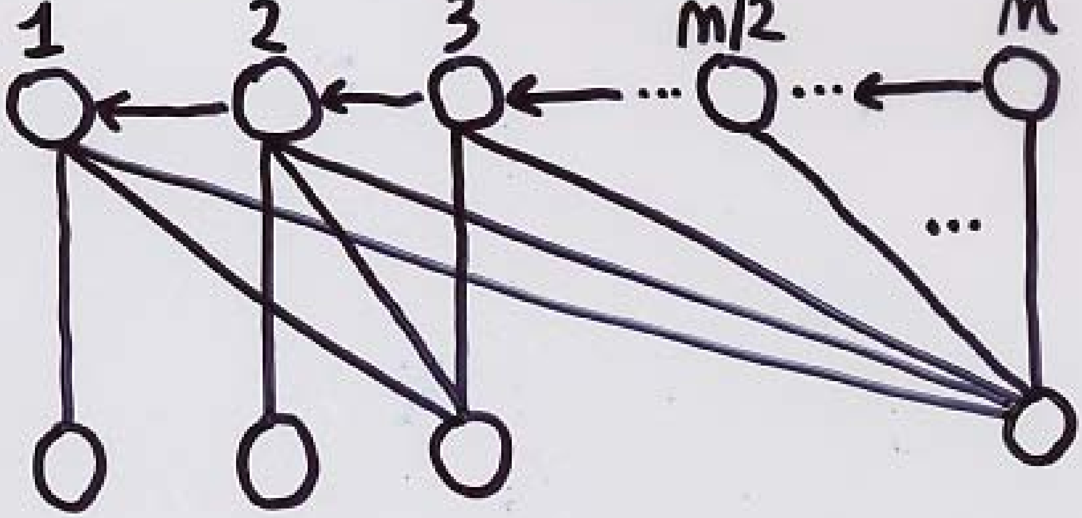

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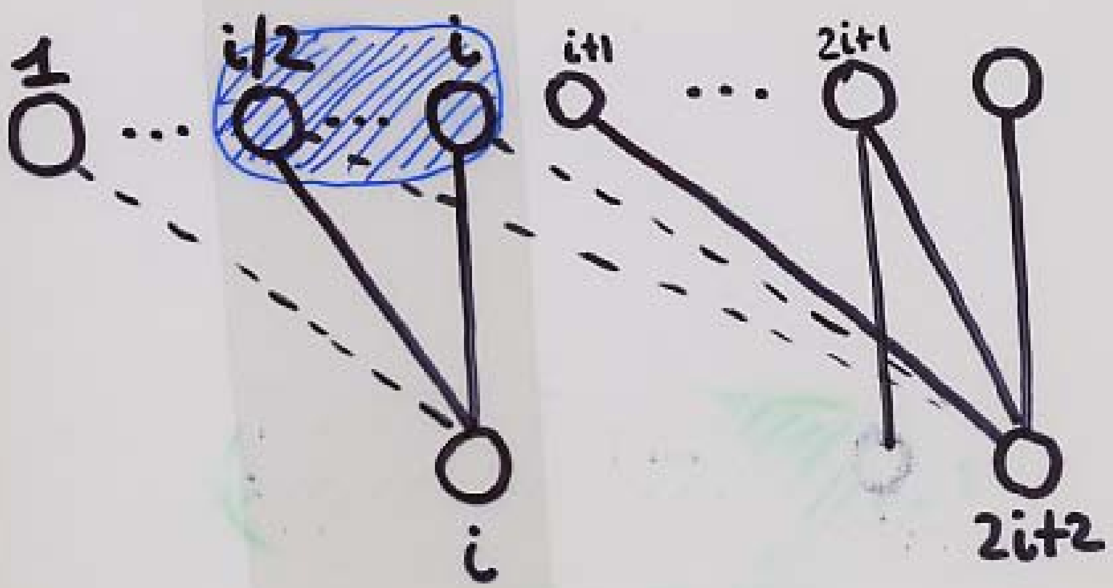

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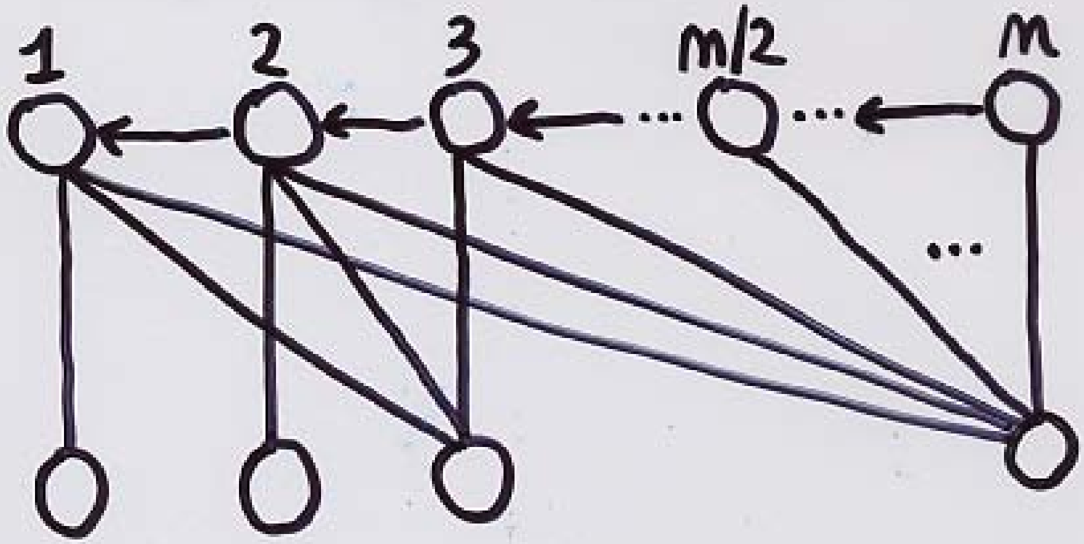

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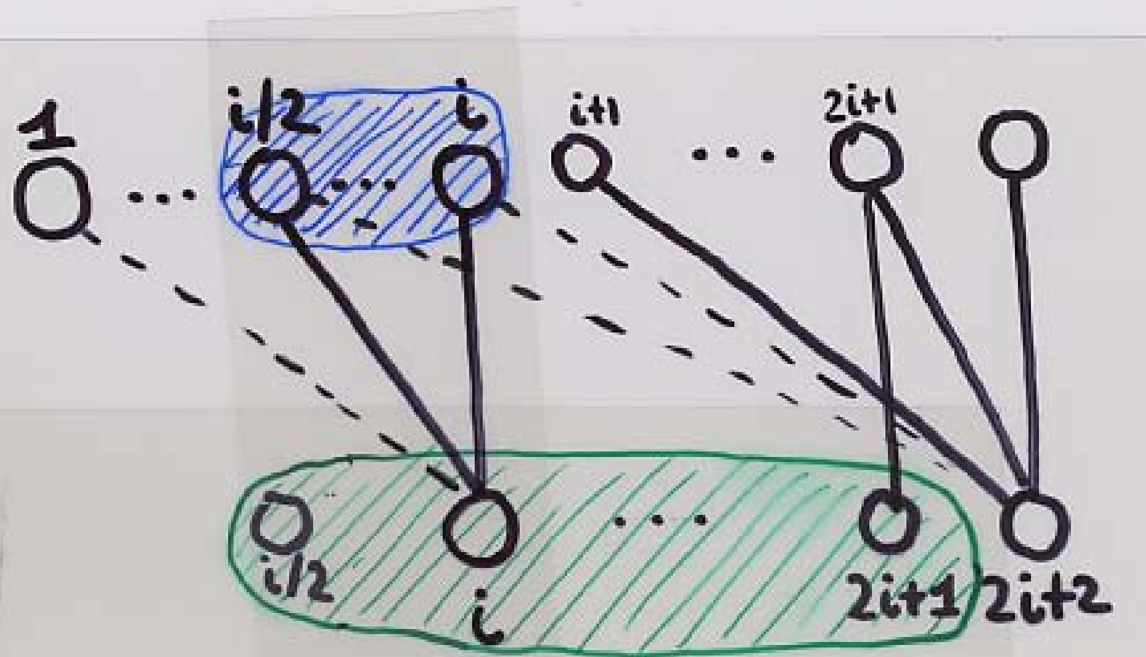

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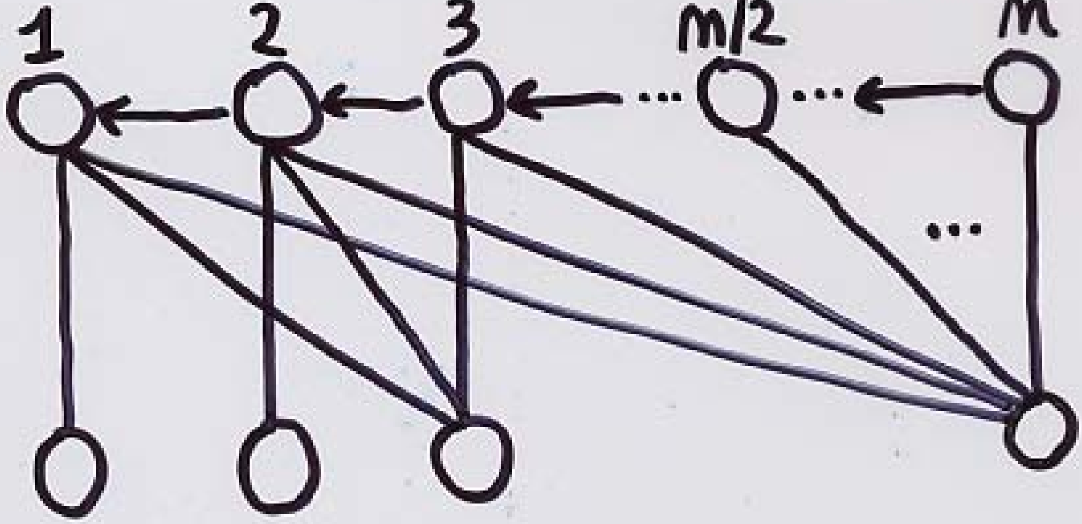

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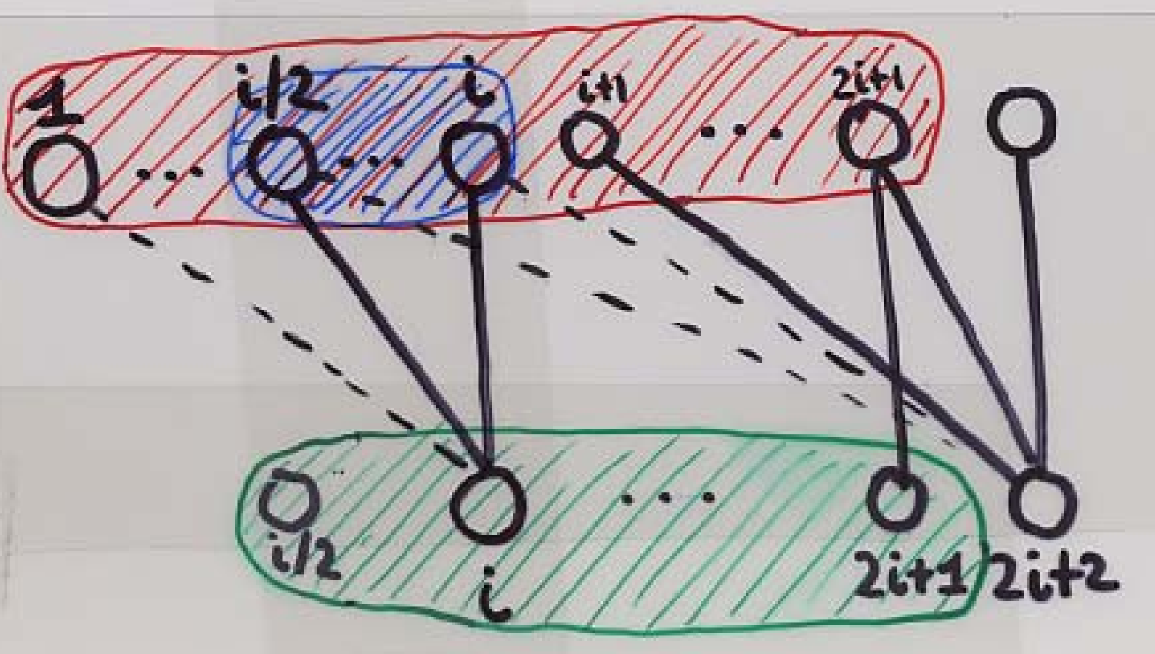


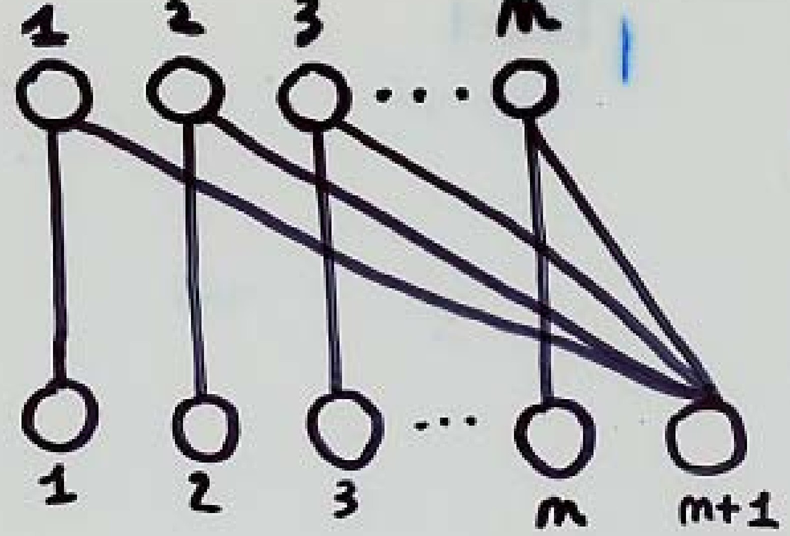
$$\frac{2i+1}{i/2} \approx 4$$

ADDING <u>FURTHER CONSTRAINTS</u> IMPROVES THE GREEDY ALGORITHM

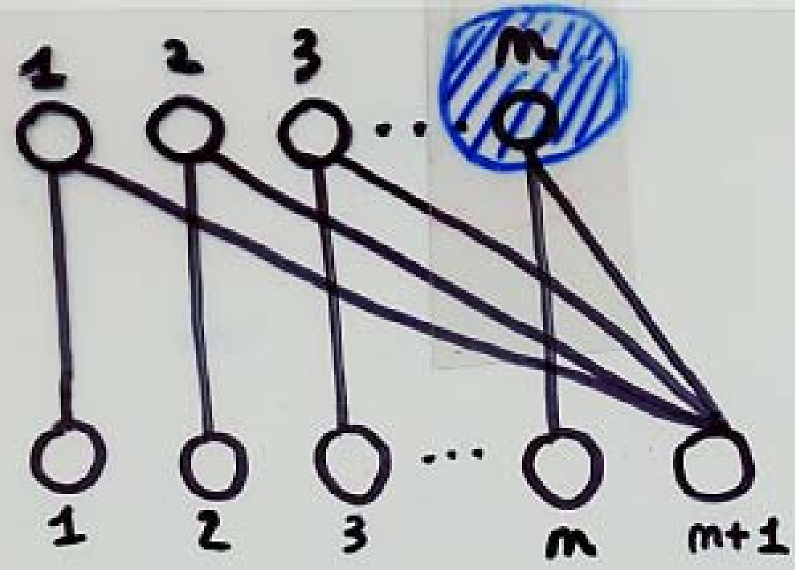
<u>OBSERVE</u>: THE ADVERSARY IS STILL USING THE ORIGINAL GRAPH

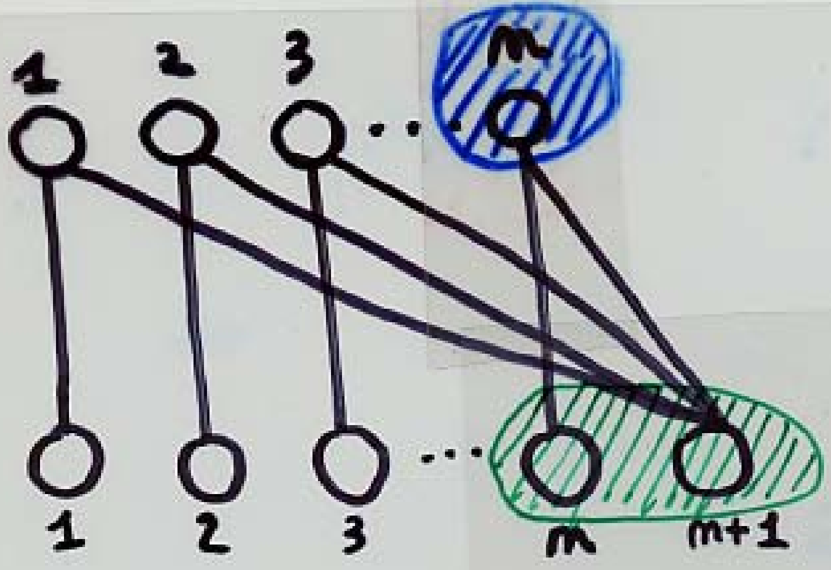
FURTHER RESULTS

- FINDING THE BEST SUBGRAPH IS NP-HARD, NO PTAS
- EFFICIENT CONSTRUCTION FOR INTERESTING CASES (HIERARCHICAL TOPOLOGIES AND OTHERS)
- SUFFICIENT CONDITIONS FOR $O(\sqrt{m})$-COMPETITIVE ALGORITHMS

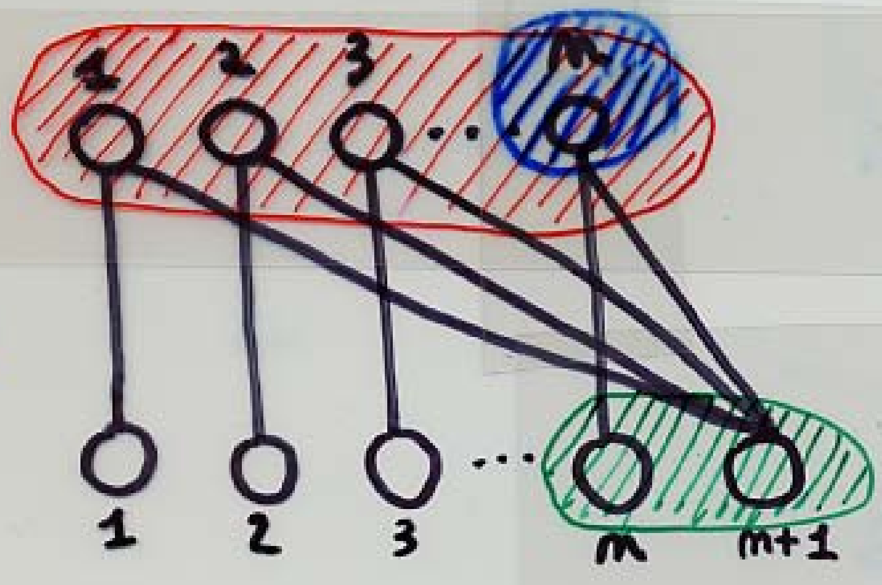


1
2
3
m
1
2
3
m
m+1

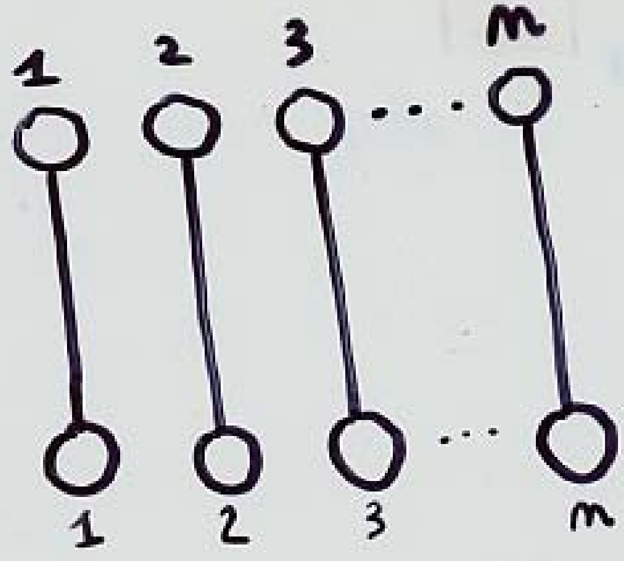

1 2 3 m
1 2 3 m m+1

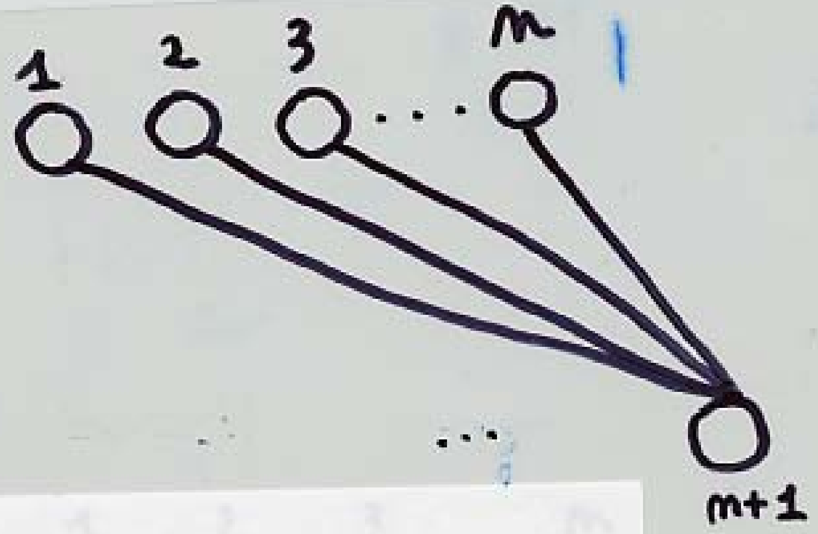


1 2 3 m
1 2 3 m m+1



$\frac{m}{1}$!!! ☹


1
2
3
m
1
2
3
m

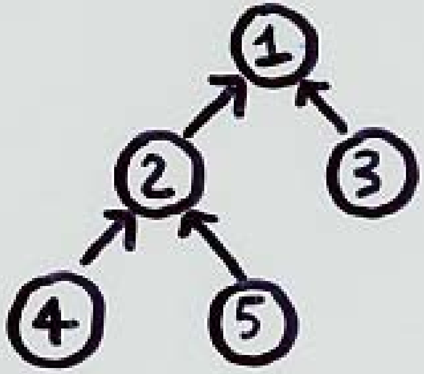

1
2
3
m
m+1

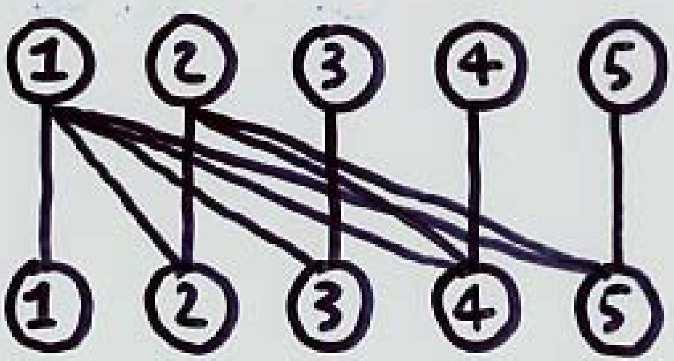
GENERAL CASE

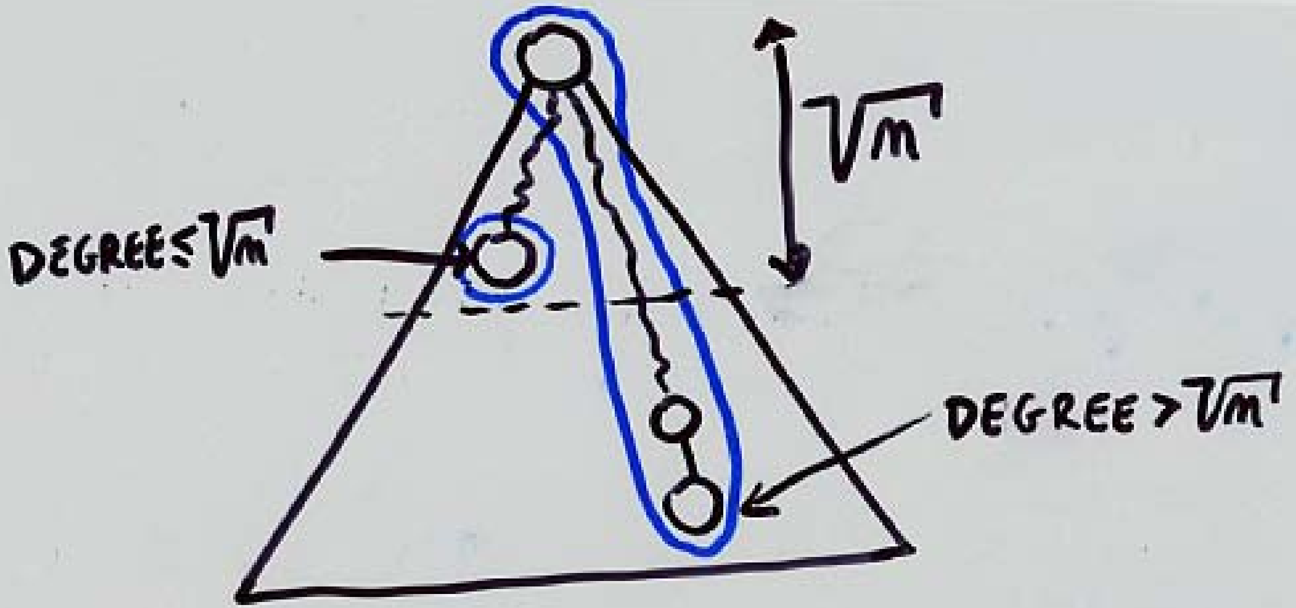
$$
\begin{tikzcd}[column sep=small]
 & \text{GRAPH } G \arrow[dl] \arrow[dr] & \\
G_H \text{ DEGREE} > \sqrt{m} \arrow[d] & & G_L \text{ DEGREE} \leq \sqrt{m} \arrow[d] \\
\text{GREEDY IS } \sqrt{m}\text{-COMPETITIVE} & & \text{MATCHING } (G_L' \subseteq G_L) \arrow[d] \\
 & & \text{SUB-GREEDY IS } \sqrt{m}\text{-COMPETITIVE}
\end{tikzcd}
$$

THEOREM. IF G HAS A MATCHING THEN SUB-GREEDY IS $(2\sqrt{m}+2)$-COMPETITIVE.

TREE HIERARCHICAL TOPOLOGIES







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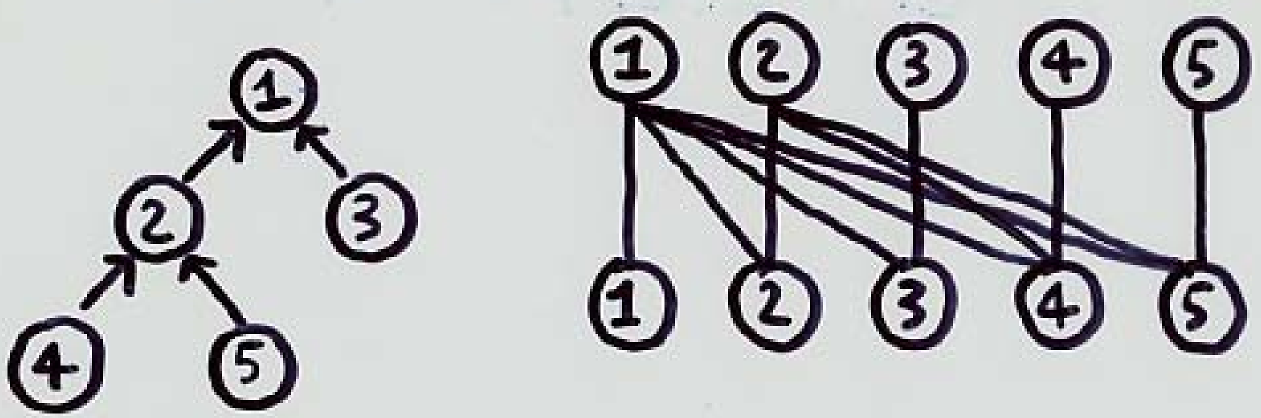

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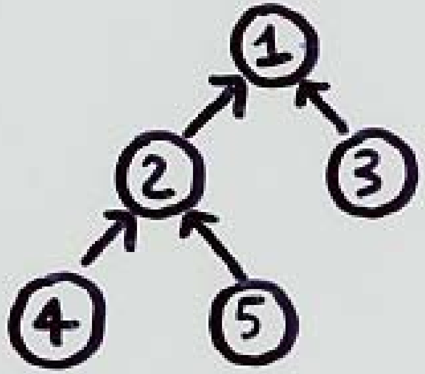

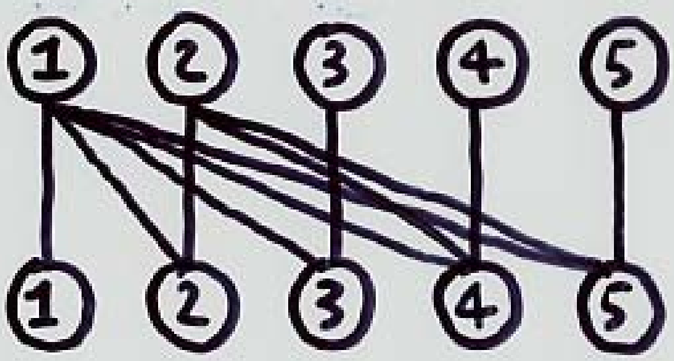

THEOREM. IF G HAS A MATCHING THEN SUB-GREEDY IS $(2\sqrt{m}+2)$-COMPETITIVE

THEOREM [BAR-NOY ET AL '99] ANY ONLINE ALGORITHM IS $\Omega(\sqrt{m})$-COMPETITIVE.

OPEN PROBLEMS

WHICH GRAPHS YIELD

- $O(\sqrt{m})$-COMPETITIVE ALGORITHMS
- OPTIMAL ALGORITHMS

APPROXIMATION ALGORITHM FOR COMPUTING THE SUBGRAPH