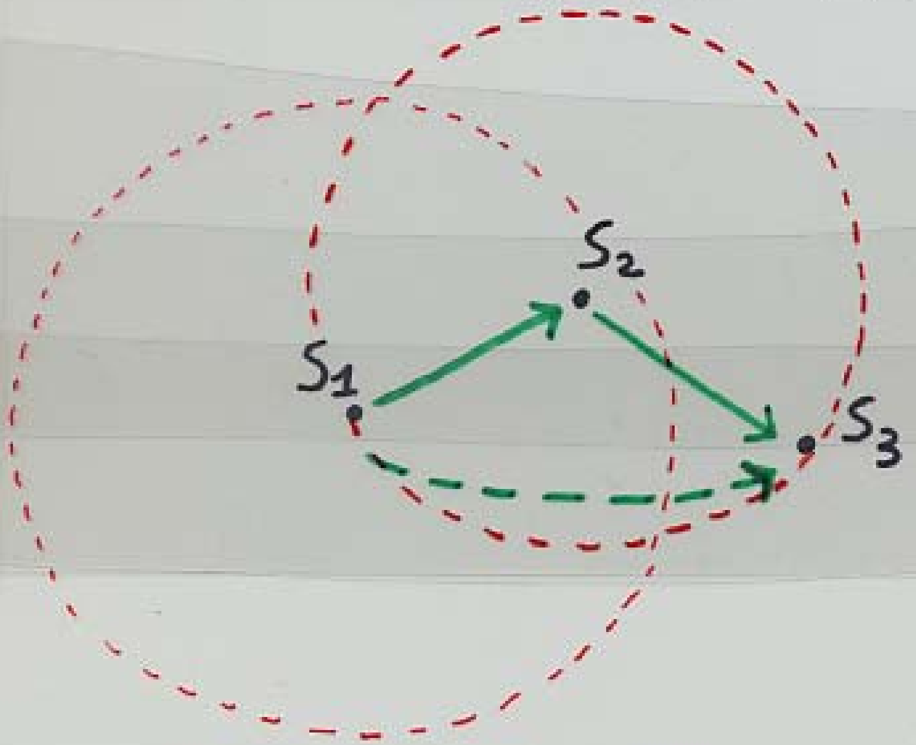
THE MINIMUM RANGE ASSIGNMENT PROBLEM ON LINEAR RADIO NETWORKS

A. CLEMENTI[1], A. FERREIRA[2], P. PENNA[1],
S. PERENNES[2], R. SILVESTRI[3]

[1] UNIVERSITY OF ROME "TOR VERGATA", MATH DEPARTMENT
[2] I3S-CNRS-INRIA SOPHIA ANTIPOLIS, MASCOTTE PROJECT
[3] UNIVERSITY OF L'AQUILA, PURE AND APPLIED MATHEMATICS DEPT.

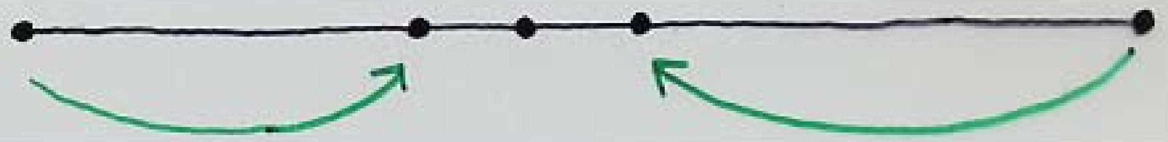
THE PROBLEM

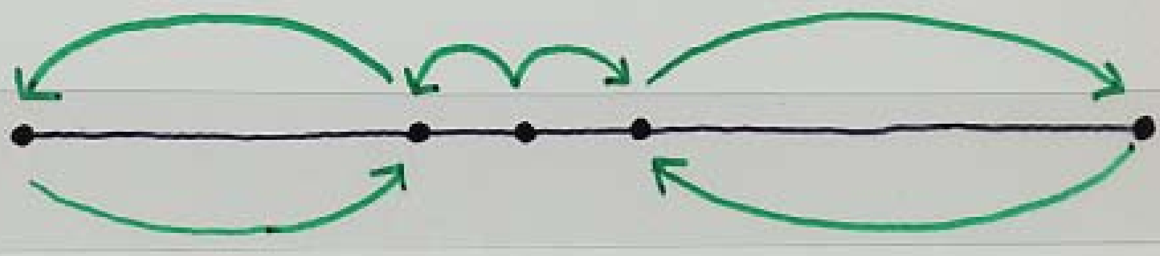
MULTI-HOP RADIO NETWORKS:

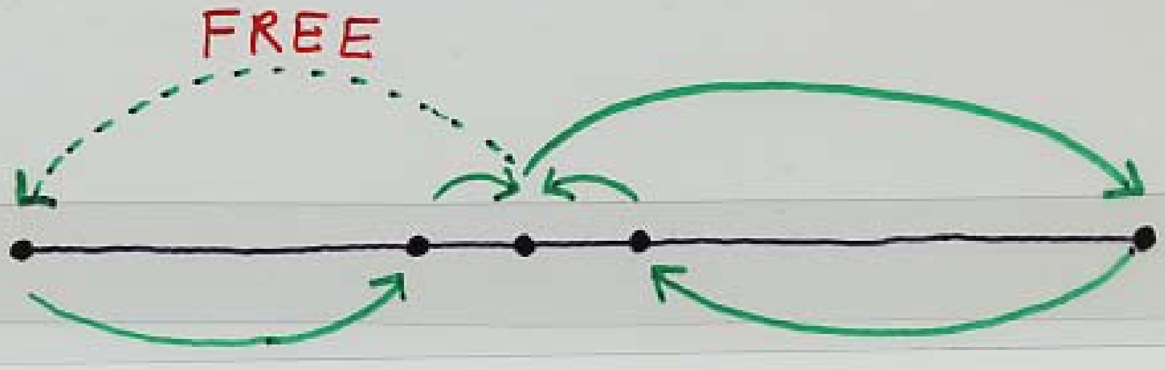


- NO INFRASTRUCTURE
- ADJUSTABLE RANGES
- MULTI-HOP TRANSMISSION

GOAL: COMMUNICATION WITH MINIMAL ENERGY






FREE

- POWER $\approx$ RANGE$^{\alpha}$, $\alpha \geqslant 2$
  (IDEALLY $\alpha = 2$)

- MULTI-HOP $\Rightarrow$ LESS ENERGY

1 HOP $\Rightarrow$ OVERALL ENERGY =

$$(n-1)^2 + (n-2)^2 + \cdots = \underline{O(n^3)}$$

$n-1$ HOPS $\Rightarrow$ OVERALL ENERGY = n

MIN RANGE R-HOPS

INSTANCE: $S = \{s_1, \ldots, s_n\} \subset \mathbb{R}^d$

SOLUTION: RANGE: $S \longrightarrow \mathbb{R}^+$ S.T.
ALL-TO-ALL COMMUNICATION
WITHIN R HOPS

MEASURE: OVERALL POWER
CONSUMPTION

$$\sum_{i=1}^{n} \text{RANGE}(s_i)^2$$

MIN RANGE R-HOPS

INSTANCE: $S = \{s_1, \ldots, s_n\} \subset \mathbb{R}^d$

HERE $d = 1$ (VEHICULAR TECHNOLOGY RADIO NETS)

SOLUTION: RANGE: $S \longrightarrow \mathbb{R}^+$ S.T.
ALL-TO-ALL COMMUNICATION
WITHIN R HOPS

MEASURE: OVERALL POWER CONSUMPTION

$$\sum_{i=1}^{n} \mathrm{RANGE}(s_i)^2$$

PREVIOUS WORKS

UNBOUNDED # HOPS:

1D	$O(m^4)$ TIME ALG. [K97]
2D	NP-HARD [C99]
3D	APX-HARD [C99]

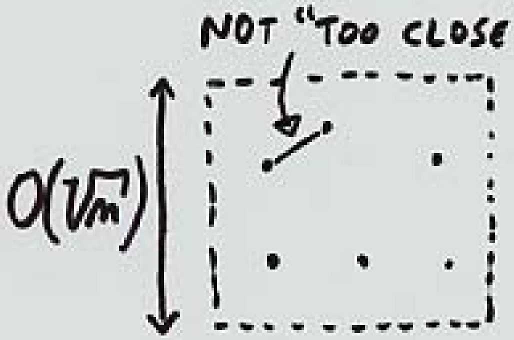
BOUNDED # HOPS:

TIGHT BOUNDS ON THE ENERGY FOR SPECIAL CONFIGURATIONS

[K97]


$\Theta(m^{\ell(R)})$, $R \in O(1)$

$\Theta(m^2/R)$, $R \in \Omega(\log m)$

[C00]


$\Theta(m^{1+1/R})$, $R \in O(1)$

[K97] KIROUSIS ET AL, STACS 97

[C99, C00] CLEMENTI ET AL, APPROX 99 STACS 00

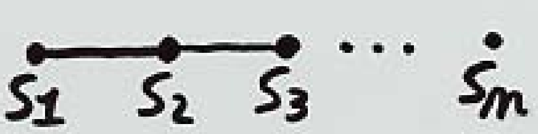
PREVIOUS WORKS

UNBOUNDED # HOPS:

1D	$O(n^4)$ TIME ALG. [K97]	
2D	NP-HARD [C99]	2-APX ALG [K97]
3D	APX-HARD [C99]	

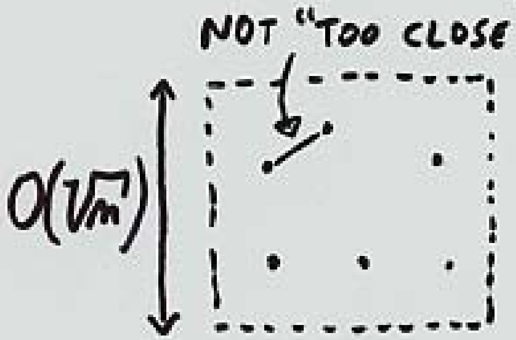
BOUNDED # HOPS:

TIGHT BOUNDS ON THE ENERGY FOR SPECIAL CONFIGURATIONS

[K97] 

$\Theta(n^{2(h)})$, $h \in O(1)$

$\Theta(n^2/h)$, $h \in \Omega(\log n)$

[C00] 

$\Theta(n^{1+1/h})$, $h \in O(1)$

[K97] KIROUSIS ET AL, STACS 97

[C99, C00] CLEMENTI ET AL, APPROX 99
STACS 00

PREVIOUS WORKS

UNBOUNDED # HOPS:

1D $O(m^4)$ TIME ALG. [K97]

2D NP-HARD [C99]

3D APX-HARD [C99]

} 2-APX ALG [K97]

BOUNDED # HOPS:

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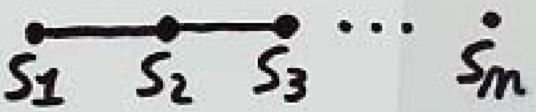
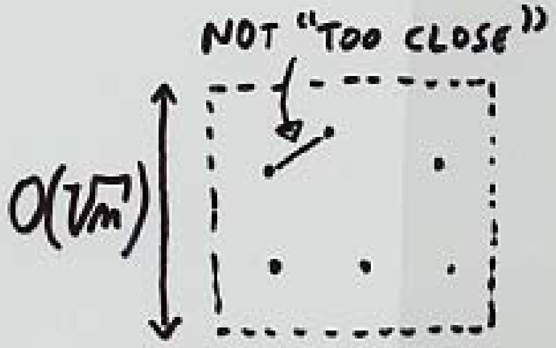
[K97] 

* $\Theta(m^{\ell(R)})$, $R \in O(1)$

$\Theta(m^2/R)$, $R \in \Omega(\log m)$

[C00] 

* $\Theta(m^{1+1/R})$, $R \in O(1)$

*: HIDDEN CONSTANT IS R !! $\Rightarrow$ R-APX ALG

[K97] KIROUSIS ET AL, STACS 97

[C99, C00] CLEMENTI ET AL, APPROX 99, STACS 00

OUR RESULTS

① ALL-TO-ONE PROBLEM $O(R \cdot n^3)$-TIME

$\Downarrow$

2-APX ALG FOR ANY R

OUR RESULTS

① ALL-TO-ONE PROBLEM $O(R \cdot n^3)$-TIME

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ANY CONFIGURATION !!!

OUR RESULTS

① ALL-TO-ONE PROBLEM $O(R \cdot n^3)$-TIME

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2-APX ALG FOR ANY R

ANY CONFIGURATION!!!

② OPT WITH BASES PROBLEM $O(R \cdot n^3)$-TIME

OUR RESULTS

① ALL-TO-ONE PROBLEM $O(R \cdot n^3)$-TIME

$\Downarrow$

2-APX ALG FOR ANY R

ANY CONFIGURATION!!!

② OPT WITH BASES PROBLEM $O(R \cdot n^3)$-TIME

$\Downarrow$

$(1+o(1))$-APX ALG FOR
WELL-SPREAD INSTANCES, $R \in O(1)$

OUR RESULTS

① ALL-TO-ONE PROBLEM $O(R \cdot n^3)$-TIME

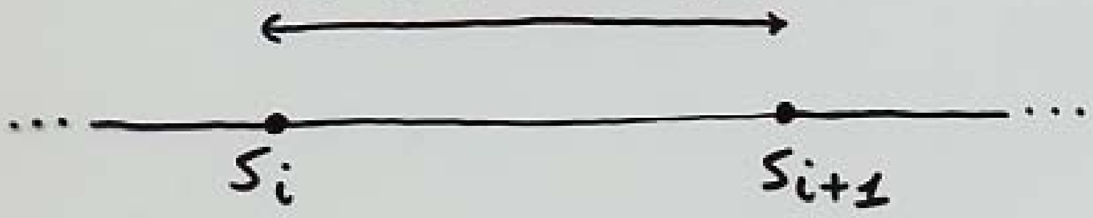
$$\Downarrow$$

2-APX ALG FOR ANY R

ANY CONFIGURATION !!!

② OPT WITH BASES PROBLEM $O(R \cdot n^3)$-TIME

$1 \leq \delta \leq$ POLYLOG(n)



OUR RESULTS

① ALL-TO-ONE PROBLEM $O(R \cdot n^3)$-TIME

$\Downarrow$

2-APX ALG FOR ANY R

ANY CONFIGURATION!!!

② OPT WITH BASES PROBLEM $O(R \cdot n^3)$-TIME

$\Downarrow$

$(1+o(1))$-APX ALG FOR
WELL-SPREAD INSTANCES, $R \in O(1)$

③ MIN RANGE 2-HOPS IS IN P

2-APX: MAIN IDEAS

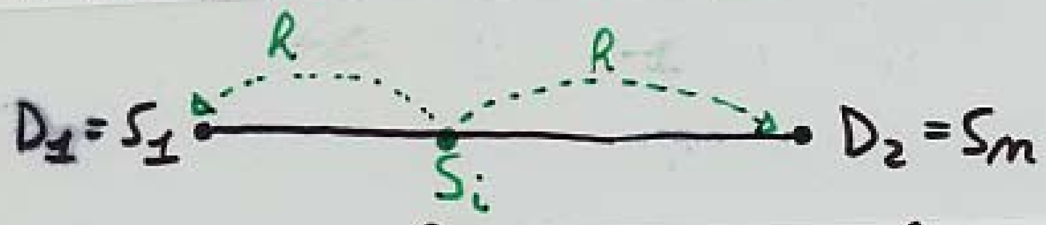
① FIX TWO DESTINATIONS:

$D_1 = S_1$ •————————• $D_2 = S_m$

$\forall S_i : \; S_i \xrightarrow{R} D_1$ AND $S_i \xrightarrow{R} D_2$

2-APX: MAIN IDEAS

① FIX TWO DESTINATIONS:



$\forall S_i : \; S_i \xrightarrow{R} D_1$ AND $S_i \xrightarrow{R} D_2$

② ALL-TO-ONE: ONLY ONE DESTINATION

2-APX: MAIN IDEAS

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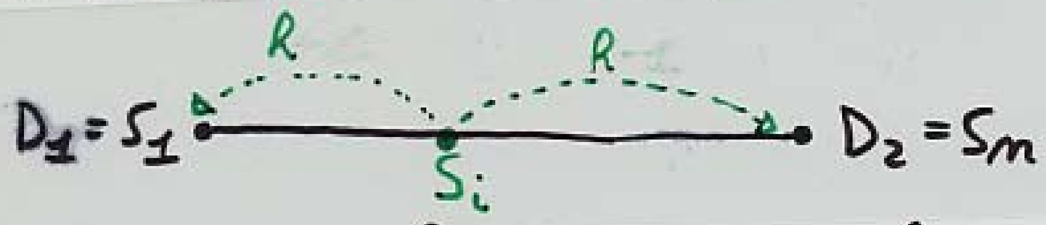

$$\forall S_i : \; S_i \xrightarrow{R-1} D_1 \text{ AND } S_i \xrightarrow{R-1} D_2$$

② ALL-TO-ONE: ONLY ONE DESTINATION

$$\forall S_i : \; S_i \xrightarrow{R-1} S_m$$

2-APX: MAIN IDEAS

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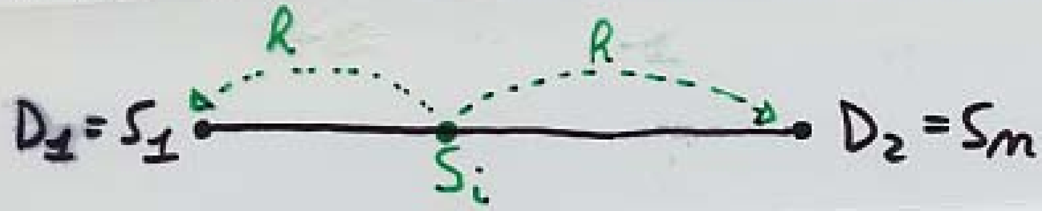


$\forall S_i : \quad S_i \xrightarrow{R} D_1$ AND $S_i \xrightarrow{R} D_2$

② ALL-TO-ONE: ONLY ONE DESTINATION

$$\forall S_i : \quad S_i \xrightarrow{R} S_m$$

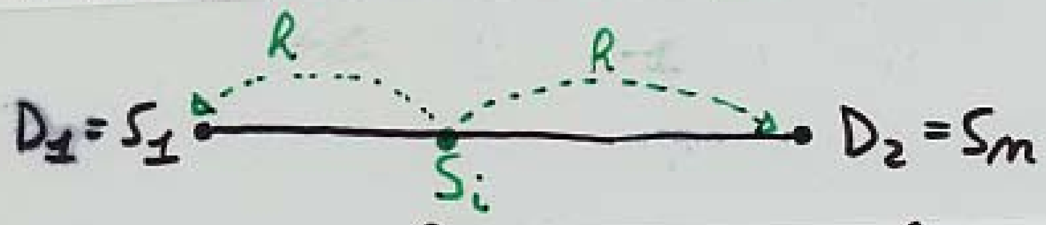
③ ALL-TO-ONE(S_1) + ALL-TO-ONE(S_m)

$$\Downarrow$$

2-APX SOLUTION

2-APX: MAIN IDEAS

① FIX TWO DESTINATIONS:



$\forall S_i : \; S_i \xrightarrow{R} D_1$ AND $S_i \xrightarrow{R} D_2$

② ALL-TO-ONE: ONLY ONE DESTINATION

$\forall S_i : \; S_i \xrightarrow{R} S_m$ EASIER!!!

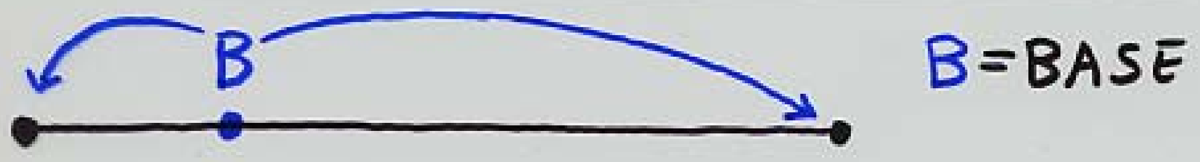
③ ALL-TO-ONE(S_1) + ALL-TO-ONE(S_m)

$\Downarrow$

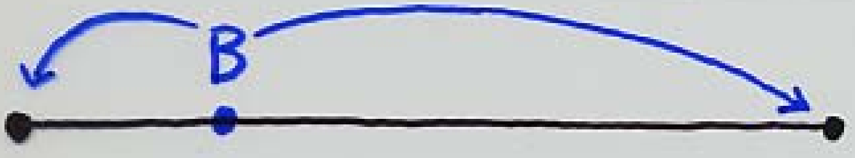
2-APX SOLUTION

BASES LOCATION PROBLEM

BASES LOCATION PROBLEM



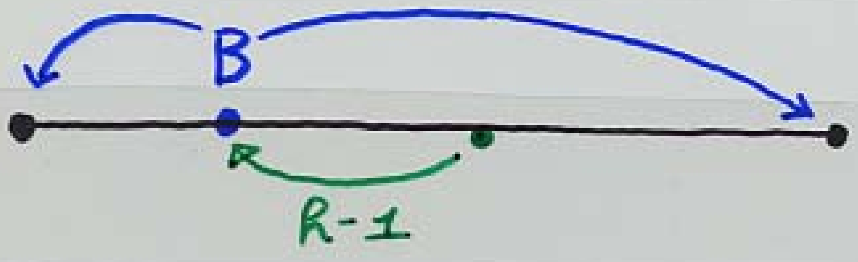
BASES LOCATION PROBLEM



B=BASE

B=DESTINATION ⇒ ALL-TO-ONE (B)

BASES LOCATION PROBLEM



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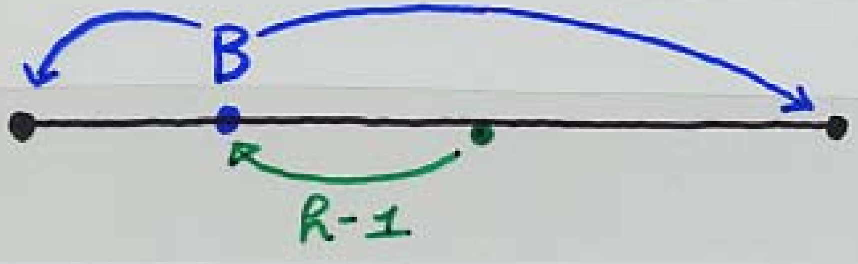

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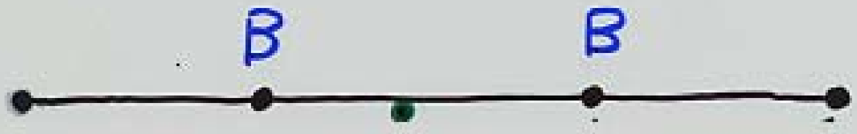

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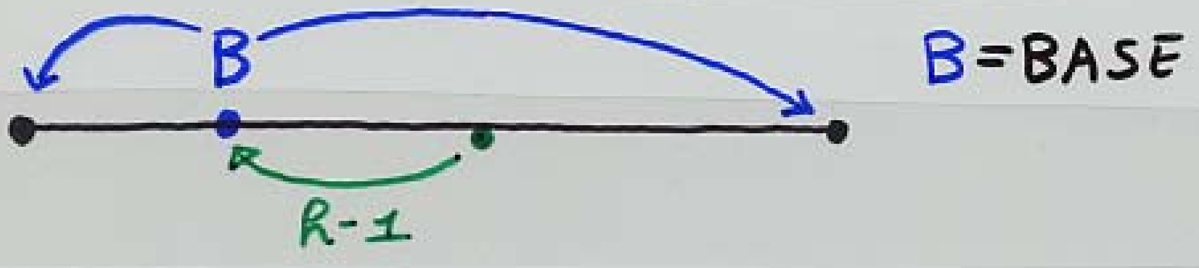

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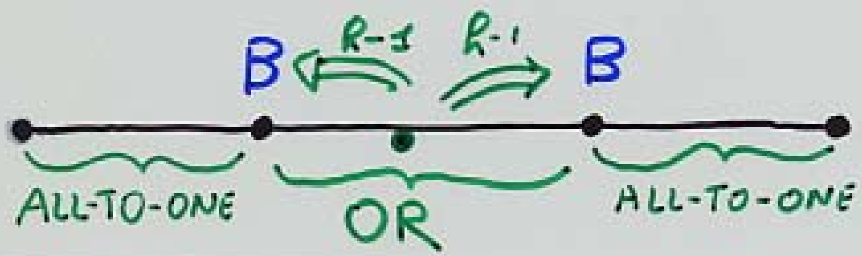

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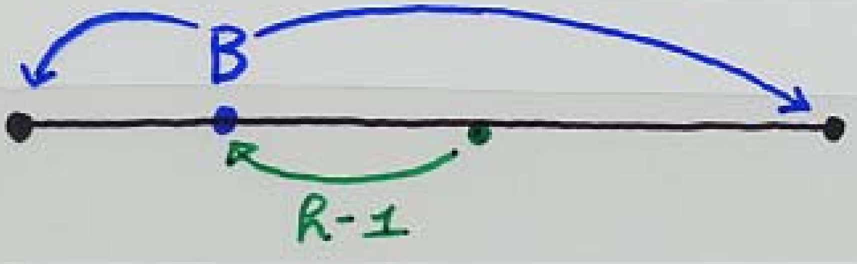

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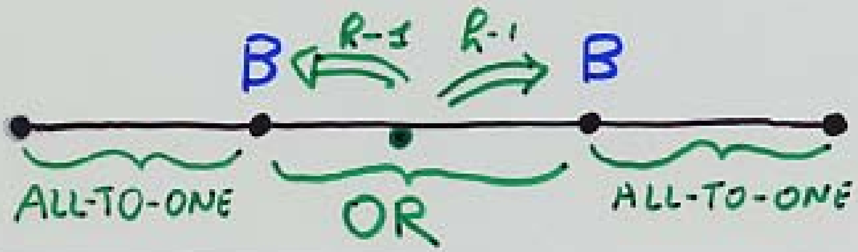

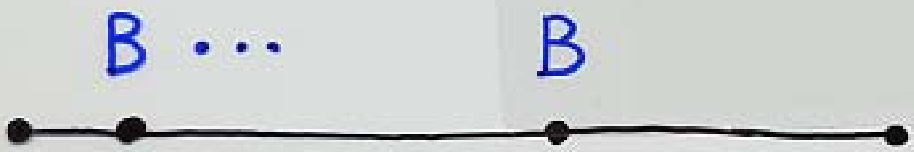

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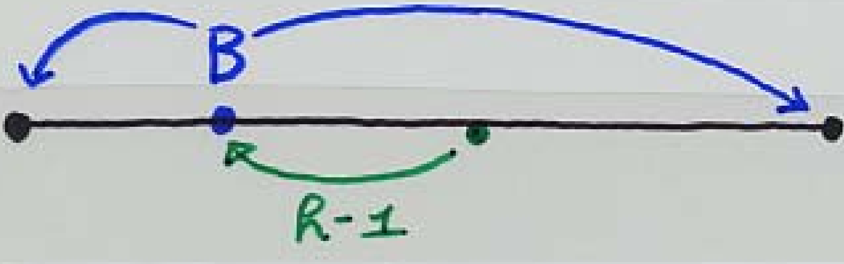

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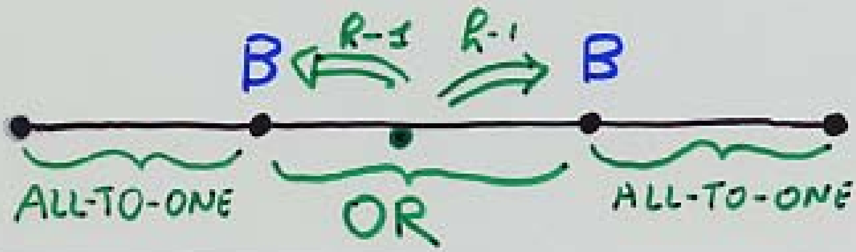

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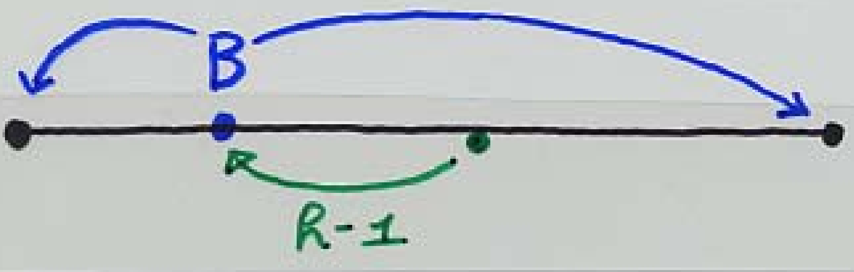

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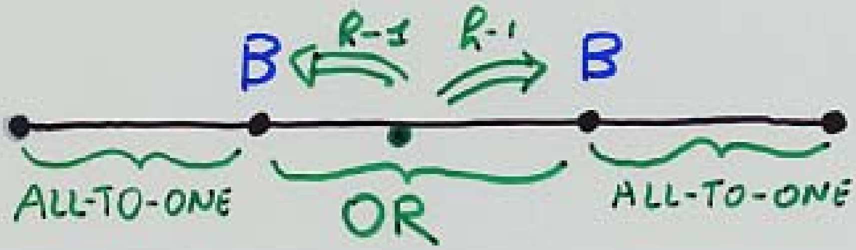
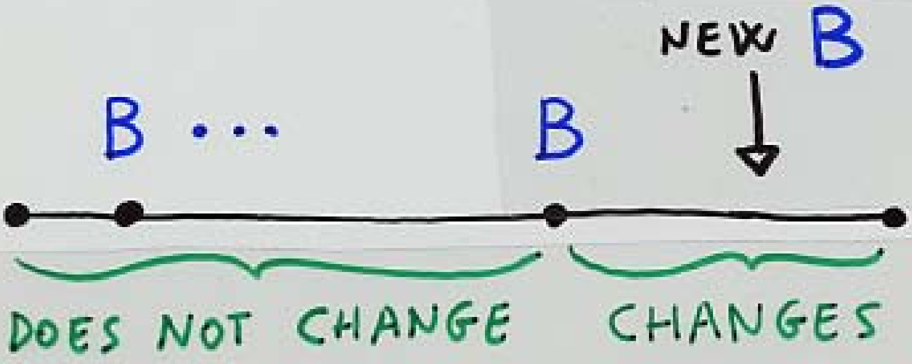
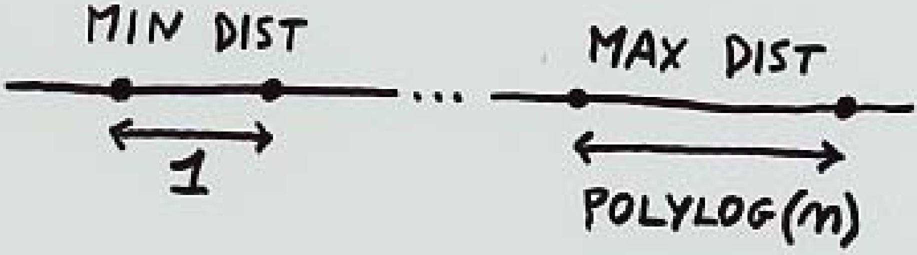




WELL-SPREAD APX ALG

WELL-SPREAD:



$$\Downarrow$$

FOR CONSTANT R
THE OPTIMUM CONTAINS MANY BASES

$$\Downarrow$$

$$\text{OPT-BASES} \leq (1+o(1)) \cdot \text{OPT}$$

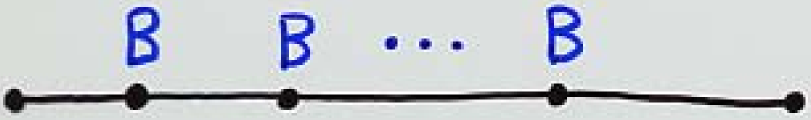
MIN RANGE 2-HOPS $\in$ **P**

IDEA: ONLY THREE OPTIMAL CONFIGURATIONS

①

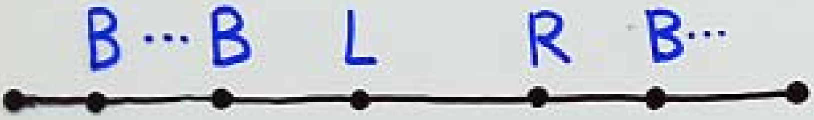
NO BASES

②



NO L,R

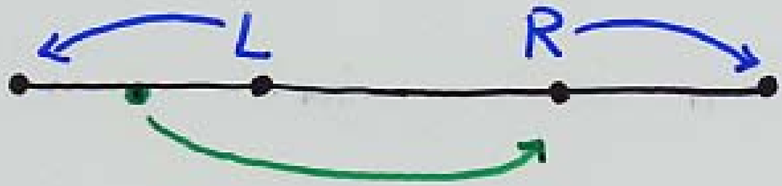
③



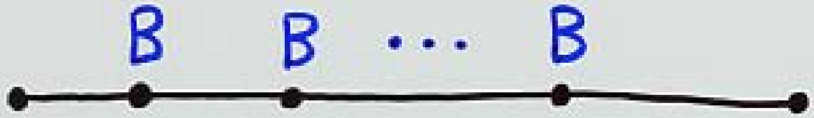
B^*LRB^*

MIN RANGE 2-HOPS ∈ P

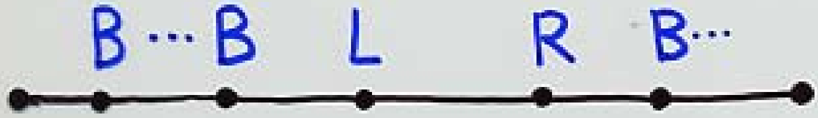
IDEA: ONLY THREE OPTIMAL CONFIGURATIONS

① 

NO BASES

② 

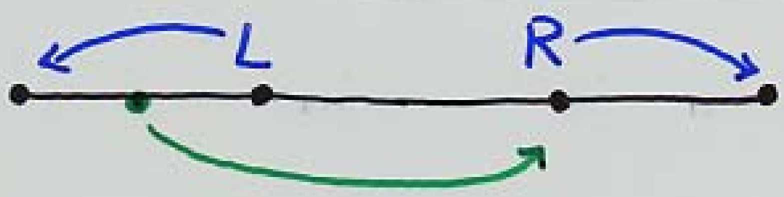
NO L,R

③ 

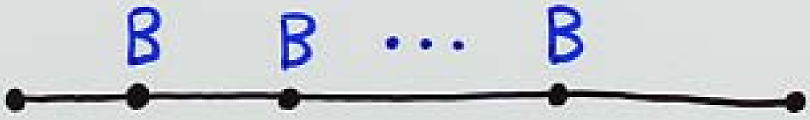
B^*LRB^*

MIN RANGE 2-HOPS $\in$ P

IDEA: ONLY THREE OPTIMAL CONFIGURATIONS

① 

NO BASES

② 

NO L,R

③ 

$B^* L R B^*$

OPEN PROBLEMS

- MIN RANGE h-HOPS $\in \mathbf{P}$?
  (FOR $h \geqslant 3$)

- 2D MIN RANGE h-HOPS $\in \mathbf{P}$?
  (FOR $h \in O(1)$, $h \in O(\log n)$, ...)

<u>NOTE</u>: NP-HARD FOR $h \in \Omega(n^{\alpha})$, $0 < \alpha < 1$