Interference Games in Wireless Networks*

Vincenzo Auletta, Luca Moscardelli, Paolo Penna, and Giuseppe Persiano

Dipartimento di Informatica ed Applicazioni "R. M. Capocelli",
Via Ponte don Melillo - I-84084 Fisciano (SA), University of Salerno, - Italy

Abstract. We present a game-theoretic approach to the study of scheduling communications in wireless networks and introduce and study a class of games that we call *Interference Games*. In our setting, a player can successfully transmit if it "*shouts strongly enough*"; that is, if her transmission power is sufficiently higher than all other (simultaneous) transmissions plus the environmental noise. This physical phenomenon is commonly known as the Signal-to-Interference-plus-Noise-Ratio (SINR).

1 Introduction

We study *Interference Games* which arise in the context of wireless communications where multiple transmissions create interference and thus unnecessary energy loss for the nodes. Each node can be regarded as a player who has her own "profit" from successfully transmitting data, and a cost proportional to the energy spent for transmitting.

The scenario in which each player of the network acts independently so to optimize her own payoff (the "net profit" given by the energy loss and the success/unsuccess in transmitting) gives rise to an interesting class of games which we call Interference Games. Unlike the classical congestion games [14], in Interference Games there is a single resource (the physical media) but each player has a number of strategies available (the transmitting power). Players essentially compete for the media and, in a single slot, at most one player can transmit successfully. Indeed, a player transmits successfully if her signal strength at the receiver is larger than the sum of the signals of all other players plus the environmental noise (see Section 2). Though transmitting with higher power is more expensive, players may strategically decide to do so because they care more about successfully transmitting. This creates a mutual interference which may result in suboptimal performance like unnecessary energy consumption and/or transmissions failures (it may be well be the case that all players transmit with high power and thus they all fail).

1.1 Our Contribution

It is natural to ask how well does the system work if players optimize their own payoff, that is, if they only care about the success of their own transmission and the energy they

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spent for it. To this aim, we study several notions of equilibria and their global efficiency. In particular, we consider *Nash equilibria* [12] where no player has an incentive to unilaterally change her strategy, *correlated equilibria* [2,3] where players have no incentive to deviate from a "suggested" strategy, and *sink equilibria* where players periodically perform best response and the whole system cycles through a "sink" consisting of a set of states of the game [6]. For all of these notions, it is possible to quantify the "system" performance in terms of *social welfare* and *fairness*. The former measures the overall "happyness" of the players, while the second one concerns how "equally" players have been treated. Finally, we consider *repeated games* [13], in which the "basic" Interference Game is played (possibly infinitely) many times. In such a context, we consider *subgame perfect equilibria* which provide a stronger solution concept compared to Nash equilibria (intuitively, the underlying "protocol" is also robust to deviations that occur for a finite amount of iterations).

We prove the following results on the existence and performance of the considered notions of equilibria for the case of a "perfectly symmetric" game in which all players valuate a successful transmission the same amount v, and they have the same set of strategies (see Section 2). We show that pure Nash equilibria do not exist if there are at least two transmission powers. Since mixed Nash equilibria and sink equilibria always exist [12,6], we consider these two notions. For two players, there exist sink equilibria with social welfare $v - k$, with k being the number of possible transmission powers, and these equilibria are also fair for odd k.

We show that every mixed Nash equilibrium has either bad social welfare or bad fairness (i.e., one of the two is equal zero). In contrast, we prove that correlated equilibria can be fair and attain a positive social welfare greater than $v - 2k$ (this improves to $v - k$ in the case of odd k). We also show optimal fair correlated equilibria for some specific games (namely, for $k \leq 3$).

Finally, we consider the case of infinitely *repeated* games with discount factor [13]. We prove that for the two-player scenario it is possible to obtain fair subgame perfect equilibria with optimal social welfare (i.e., $v - 1$). The result holds for the case in which every player knows only if her previous transmissions were successful or not.

1.2 Related Work

Fiat et al. [4] study contention resolution protocols for selfish agents aiming at accessing a broadcast channel. They focus on the scenario in which each player has one packet to transmit and she can choose either to transmit or not to transmit at each time slot (that is, each player has two possible strategies). They analyze the well known *Aloha* protocol and provide a new protocol being a Nash equilibrium for the game and having better performances (in terms to transmission delays) with respect to *Aloha*.

Adlakha et al. [1] study Bayesian Interference Games in a wireless scenario in which players select a power profile over the available bandwidth to maximize their own data rate (measured via Shannon capacity). They analyze Nash equilibria of the incomplete information game in which players are unaware of the interference they cause to the other ones.

In Timing Games [8,9,10] two players must decide when to make a single move at some time between 0 and T. The payoffs of the players usually depend on which player moves first and/or the time that she moves. Though Timing Games can be seen as special Interference Games, in Timing Games the payoffs are always positive which is not true for Interference Games. This determines a different structure of equilibria.

Scheduling wireless communications under the Signal-to-Interference-plus-Noise-Ratio (SINR) model have been studied in [7,11].

2 Model and Definitions

In the Signal-to-Interference-plus-Noise-Ratio (SINR) model (see for instance [7,11]), a node α is successful in transmitting if and only if

$$\frac{p_\alpha / d_\alpha^a}{\text{Noise} + \sum_{\beta \neq \alpha} (p_\beta / d_\beta^a)} \geq B, \tag{1}$$

where d_α is the distance of node α from the receiver and p_α is the power of node α's transmission.

We study the SINR model from a game theoretical point of view and introduce a class of games which we call *Interference Games*. There are n players corresponding to the nodes aiming to communicate. A strategy of a player α is an integral power transmission level in $\{0, 1, \ldots, k\}$ and all players have the same set of strategies. Moreover, we denote by v how much a successful transmission is worth to a player (we assume this value to be the same for all players).

Given a strategy profile or *state* $s = (s_1, \ldots, s_\alpha, \ldots, s_n)$ of the game, player α is *successful* if s_α is larger than the sum of all other s_i's. Notice that s_α is the power of α and the condition for being successful corresponds to the case in which all nodes are at the same distance from the receiver, $\text{Noise} > 0$, and $B = 1$ (see Equation 1). The *utility* or *payoff* $u_\alpha(s)$ of player α depends on her power consumption and the fact that her transmission is successful or not. Namely, if in s player α is successful and has used power p_α, then her payoff is $u_\alpha(s) = v - p_\alpha$. Otherwise, if in s player α is not successful and has used power p_α, then her payoff is $u_\alpha(s) = -p_\alpha$. If we deal with probabilistic choices, we are interested in the expected utility. Each players aims to maximize her own (expected) utility.

We consider the *social welfare* function $SW(s) = \sum_{\alpha \in N} u_\alpha(s)$ that is the sum of the payoff's of all players. The *fairness* of a state s is defined as the ratio between the minimum and the maximum (expected) utility of players; i.e., $\frac{\min_{\alpha \in N} u_\alpha(s)}{\max_{\alpha \in N} u_\alpha(s)}$; if the utilities of all the players are 0, the fairness is defined equal to 1. Moreover, we call *fair* a state with fairness equal to 1, and *unfair* a state with fairness equal to 0.

We now review the equilibrium notions that we use in this paper. A *pure Nash equilibrium* is a state in which no player can obtain a higher utility by changing her strategy, given the strategies of the other players. In a *mixed Nash equilibrium* we consider players picking a strategy independently according to some probability distribution (each player decides her own distribution). In a mixed Nash equilibrium no player can improve her expected payoff by changing her probability distribution, given the probability distributions of the other players. In *correlated equilibria* a "mediator" picks a state s according to some probability distribution and "suggests" strategy s_α to each player α.

Each player α is only aware of her suggested strategy and of the probability distribution used to pick the state. A probability distribution over the set S of all states is a correlated equilibrium if no player can improve her expected payoff by replacing her suggested strategy with a different one, given that the other players follow the suggested strategy (note that the expected payoff of α is *conditional* to the fact that player α has been suggested some strategy s_α). In *sink equilibria* we consider a so called state graph in which every node corresponds to a state of the game and there is a directed edge from s to s' if there is a player α such that $u_\alpha(s) < u_\alpha(s')$ *and* state s' is obtained from s by changing strategy s_α with some other strategy s'_α. Intuitively, edges corresponds to best response of some player and, in a sink equilibrium, players moves will "cycle" through some connected component (when the component has only one node we have a pure Nash equilibrium). More formally, sink equilibria are the strongly connected components of the state graph. Let Q be a sink equilibria and let $\pi : Q \to \mathbb{R}^+ \cup \{0\}$ be the steady state distribution of the random walk over states $q \in Q$ of the sink equilibrium. The (expected) social welfare of Q is the expected social value of states given by the steady distribution of the random walk over its states; i.e. $SW(Q) = \sum_{q \in Q} \pi(q) SW(q)$.

3 A Simple Interference Game

In this section we analyze a simple Interference Game characterized by $n = 2$ players and $k = 2$. The game is perfectly symmetric and a player α is successful if and only if $p_\alpha > p_\beta$, where β is the other player. Despite its simplicity, we can already derive some indications from this simple game. The utility matrix is

	0	1	2
0	$0, 0$	$0, v-1$	$0, v-2$
1	$v-1, 0$	$-1, -1$	$-1, v-2$
2	$v-2, 0$	$v-2, -1$	$-2, -2$

We start by proving that this simple game has no pure Nash equilibrium.

Theorem 1. *For any $k \geq 2$, the Interference Game has no pure Nash equilibrium, even for two players.*

Proof. Observe that the best response for a player to strategy $x < k$ of the other player is strategy $x + 1$ and the best response to strategy k is strategy 0. Therefore, the only possible pure equilibria are $(0, k)$ or $(k, 0)$. Since the best response to strategy 0 is strategy 1, such states are Nash equilibria only if $k = 1$. Similar arguments apply for the case of $n > 2$ players.

We now turn our attention to sink equilibria.

Theorem 2. *The Interference Game with $n = 2$ and $k = 2$ has a unique sink equilibrium with social welfare $v - 2$ and fairness* 1.

Proof. By recalling the above considerations on the best response moves, there exists a unique sink equilibrium given by the cycle:

$$(0, 1), (2, 1), (2, 0), (1, 0), (1, 2), (0, 2), (0, 1).$$

Since the equilibrium is a cycle, the steady distribution of the random walk is the uniform one, and it is easy to check that its social value is $v-2$. Moreover, since for each state in the sink also its symmetric state is present, the fairness of the equilibrium is 1.

We continue our study by analyzing mixed Nash equilibria.

Theorem 3. *The Interference Game with $n=2$ and $k=2$ has a mixed Nash equilibrium with social welfare 0 and fairness 1.*

Proof. The equilibrium corresponds to the probability distribution $q=(q_0,q_1,q_2)$ with $q_0=q_1=1/v$. To see that this is a Nash equilibrium, let $u_\alpha^{(q)}(i)$ be the payoff of player α when it plays strategy i, given that the other one plays according to probability distribution q. Clearly $u_\alpha^{(q)}(0)=0$, while

$$u_\alpha^{(q)}(1)=q_0(v-1)-q_1-q_2=q_0v-1=0.$$

Similarly

$$u_\alpha^{(q)}(2)=q_0(v-2)-q_1(v-2)-2q_2=(q_0+q_1)v-2=0.$$

Since the payoff is constant for all three strategies, when both players play according to the probability distribution q, none has an incentive in unilaterally deviating. That is, q is a Nash equilibrium and the payoff of each node is 0; thus, also the social welfare is 0 and the equilibrium is fair.

We conclude the study of the case $n=2$ and $k=2$ by showing the best possible correlated equilibrium X, and proving that its social welfare is very close to the optimum. Each player receives a suggestion on the power to use for the transmission. We denote by $x(i,j)$ the probability that the first and the second players are suggested to transmit at power i and j, respectively. We will consider only symmetric distributions, that is, distributions for which $x(i,j)=x(j,i)$ that thus give fair correlated equilibria. We denote by q_{ij} the probability that player 2 receives suggestion j given that player 1 has received suggestion i, that is $q_{ij}=\frac{x(i,j)}{\sum_h x(i,h)}$. Since transmission of player 1 at power i is successful if and only if player 2 transmits at a power $j<i$, we have that the probability $Pr[j|i]$ that player 1 is successful at power j given that he was suggested to transmit at power i is equal to

$$Pr[j|i]=\sum_{h<j} q_{ih}$$

and the expected payoff $u_\alpha[j|i]$ of player α when transmitting at power j, given that he was suggested to transmit at power i, is equal to $vP_i[j|i]-j$. The definition of correlated equilibrium is that $u_\alpha[i|i]\geq u_\alpha[j|i]$.

Theorem 4. *For the Interference Game with $n=2$ and $k=2$, the optimal symmetric correlated equilibria has social welfare $v-2+\frac{v}{v^2-2}$.*

Sketch of proof. The following matrix turns out to be an optimal correlated equilibrium:

$$X := \begin{bmatrix} 0 & \frac{v-1}{v^2-2} & \frac{v^2-3v+2}{2v^2-4} \\ \frac{v-1}{v^2-2} & 0 & \frac{v-2}{2v^2-4} \\ \frac{v^2-3v+2}{2v^2-4} & \frac{v-2}{2v^2-4} & 0 \end{bmatrix}$$

thus proving the theorem.

4 Two Players and Arbitrarily Many Strategies

While for $k = 2$ the only Nash equilibrium has social welfare 0, it turns out that when k is odd there are Nash equilibria whose social welfare is strictly positive.

Theorem 5. *For every odd k, there exists a mixed Nash equilibrium with social welfare* $v - k$.

Sketch of proof. The following matrix is a mixed Nash equilibrium:

$$CF := \begin{bmatrix} 0 & \frac{2}{v} & 0 & \frac{2}{v} & 0 & \cdots & \frac{2}{v} & 0 & 1-\frac{k-1}{v} \\ 1-\frac{k-1}{v} & 0 & \frac{2}{v} & 0 & \frac{2}{v} & \cdots & 0 & \frac{2}{v} & 0 \end{bmatrix}.$$

Since CF is an unfair Nash equilibrium, we next investigate the existence of fair equilibria.

Theorem 6. *For any Interference Game, there exists a unique (fair) fully mixed Nash equilibrium, that is, a Nash equilibrium in which every player assigns nonzero probability to every strategy. Moreover, every fair Nash equilibrium has social welfare* 0.

Sketch of proof. In a fully mixed equilibrium, strategy 0 is in the support of every player which implies that the expected payoff of every player must be 0. Calculations show that the condition for having a Nash equilibrium impose that the probability distribution of each player is $q = (\frac{1}{v}, \frac{1}{v}, \ldots, \frac{1}{v}, 1 - \frac{k}{v})$.

At Nash equilibrium, at least one player must have 0 in her support. Thus, in every fair Nash equilibrium 0 is in the support of all players and therefore the social welfare must be 0.

Correlated equilibria can be both fair and achieve good social welfare:

Theorem 7. *For any Interference Game there exists a fair correlated equilibrium with social welfare greater than* $\max(0, v - 2k + 1)$.

Sketch of proof. We modify the joint probability distribution of the Nash equilibrium given in the proof of Theorem 6 and obtain a correlated equilibrium given by the following matrix:

$$C = \begin{bmatrix} 0 & c2\lambda^2 & \cdots & c\lambda^2 & c\lambda(1-k\lambda) \\ 2c\lambda^2 & 0 & \cdots & c\lambda^2 & c\lambda(1-k\lambda) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ c\lambda^2 & c\lambda^2 & \cdots & 0 & c(\lambda(1-k\lambda)+\lambda^2) \\ c\lambda(1-k\lambda) & c\lambda(1-k\lambda) & \cdots & c(\lambda(1-k\lambda)+\lambda^2) & 0 \end{bmatrix}$$

where $\lambda = 1/v$ and c is a suitable constant such that C is a probability distribution.

The social welfare can be further improved for even k:

Theorem 8. *For any Interference Game with k even there exists a fair correlated equilibrium with social welfare at least $v - k$. Moreover, for $k = 3$ there exists an optimal symmetric Correlated Equilibrium with social welfare $v - 3 + \frac{4v^2-11v+7}{v^3-v^2-4v-5}$.*

We next generalize the sink equilibria described in Section 3. The main difference is that for odd k there exist two sink equilibria both with fairness less than 1.

Theorem 9. *The Interference Games with $n = 2$ and k even have a unique sink equilibrium with social welfare $v-k$ and fairness 1. The Interference Games with $n = 2$ and k odd have two sink equilibria with social welfare $v-k$ and fairness $\frac{(2k-2)v-2k^2+2}{(2k+6)v-k^2-2k-1}$.*

Sketch of proof. For $k = 3$, there are *two* sink equilibria:

$$(0, 1), (2, 1), (2, 3), (0, 3), (0, 1) \text{ and } (1, 0), (1, 2), (3, 2), (3, 0), (1, 0).$$

In each of them, one player has expected utility $\frac{v}{4} - 1$, and the other one $\frac{3}{4}v - 2$. Therefore, the fairness is $\frac{v-4}{3v-8}$. A similar argument generalizes to any even k.

5 Arbitrarily Many Players

The following theorem extends the results on Nash equilibria for two players given in Section 5.

Theorem 10. *There exists a fair Nash equilibrium with $n \geq 3$ players with social welfare equal to 0. Moreover, if k is odd, there exists an unfair Nash equilibrium with $n \geq 3$ players with social welfare equal to $v - k$.*

Sketch of proof. It is possible to show that, given a Nash equilibrium for the case of two players and in which at least one player having expected utility 0, it is possible to obtain a Nash equilibrium for $n \geq 3$ players by adding $n - 2$ players playing strategy 0 with probability 1. The theorem thus follows from the results on two players (Theorems 5-6).

Correlated equilibria achieve both fairness and good social welfare:

Theorem 11. *For any n and for odd k there exists a fair Correlated Equilibrium with social welfare $v - k$.*

6 Repeated Interference Games

In the repeated interference game, the same interference game is played (infinitely) many times and, at each repetition i, player α accumulates a new payoff $\delta^i \cdot u_\alpha(s^{(i)})$, where $s^{(i)}$ are the strategies played at repetition i and $\delta < 1$ is the discount factor. A simple protocol for two players consists in alternating transmissions, with the transmitting player using power 1; Every deviation from this results in a “punishment” phase in which both players transmit with maximal power for prescribed amount of time steps; Deviations from the punishment phase will “restart” of the punishment phase itself. This results in an optimal subgame perfect equilibrium:

Theorem 12. *For every v and k there exists $\overline{\delta} < 1$ such that the following holds. For any $\delta > \overline{\delta}$, the repeated Interference Game with v and k and discount factor δ has a fair subgame perfect equilibrium with expected payoff profile* $((v-1)/2, (v-1)/2)$.[1]

The main idea is that if a player deviates from this punishment phase, this can be detected by the other player who sees that her transmission is successful (deviations from the "non-punishment" phase are detected because of transmission failure). This is sufficient for applying the result in [5].

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[1] Every two rounds the "non-discounted" payoff of each player is $v - 1$.