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On the approximability of the range assignment problem on radio networks in presence of selfish agents[☆]

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Abstract

We consider the *range assignment* problem in ad-hoc wireless networks in the context of *selfish agents*: A network manager aims to assigning transmission ranges to the stations in order to achieve strong connectivity of the network within a minimal *overall power consumption*. Station is not directly controlled by the manager and may refuse to transmit with a certain transmission range because it might be costly in terms of power consumption.

We investigate the existence of payment schemes which induce the stations to follow the decisions of a network manager in computing a range assignment, that is, *truthful mechanisms* for the range assignment problem. We provide both positive and negative results on the existence of truthful VCG-based mechanisms for this NP-hard problem. We prove that (i) in general, every polynomial-time truthful VCG-based mechanism computes a solution of cost far-off the optimum, unless P = NP and

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(ii) there exists a polynomial-time truthful VCG-based mechanism achieving constant approximation for practically relevant, still NP-hard versions, i.e., the metric and the well-spread case.

Keywords: Algorithmic mechanism design; Energy consumption in wireless networks; Approximation algorithms

1. Introduction

Ad-hoc wireless networks offer the possibility of communicating without any fixed infrastructure. Indeed, each station is a radio transmitter/receiver and communication between two stations that are not within their respective transmission ranges can be achieved by *multi-hop* transmissions: A set of intermediate stations cooperates with the source node and forwards the message to its destination. Due to the limited power of the stations, multi-hop transmissions are, in general, unavoidable. Moreover, they often result into a significant reduction of the overall energy required by the communication. This is accomplished by suitably varying the transmission ranges of the stations depending on the environmental conditions and on the relative positions of the stations.

While ad-hoc networks have been adopted in the past as an alternative to classical wired networks (e.g., whenever the use of wired connection was too expensive or simply impossible), the availability of low-cost hardware and the flexibility provided by ad-hoc networks opens new possibilities. In particular, this technology may now be applied to build up communication networks without being dependent on any "external" entity imposing its own will (e.g., governments, internet providers, private companies).

Unfortunately, the inherent *self-organized* nature of ad-hoc networks poses new problems since (i) stations are not under the control of a central authority; (ii) a station transmitting with a certain range incurs a cost proportional to the energy required and so (iii) stations may decide to not to follow the "protocol" because it is not convenient.

The impact of the *energy consumption* issue is twofold:

Social cost: The overall energy required to communicate represents a significant factor in the electromagnetic pollution, resource consumption and, in general, it has a tremendous impact on the environment. When adopted on a large scale, these factors become a central issue in the design of ad-hoc networks.

Selfish behavior: Due to the limited battery capacity of mobile devices, each node of the network aims at transmitting with range as small as possible. Thus, it is not reasonable to assume an *altruistic* behavior of the nodes in providing their battery resources to forward somebody else's messages.

These two aspects are somewhat in conflict with each other. On the one hand, without cooperation, communication is possible only at very high cost: Every station should be able to reach all other stations.[3] On the other hand, solutions that optimize the *social cost*, expressed as the overall power consumption, are based on cooperation. Resolving this conflict is a key point for the development of ad-hoc wireless networks.

[3] Interestingly, this would also result in high cost for the single station.

As for the social cost, a fundamental problem is how to assign transmission ranges to the stations so that (i) every station i can reach any other station by using intermediate ones and (ii) the overall energy is minimized. This optimization problem, usually denoted as the *minimum range assignment problem* [14], turns out to be a hard problem because of two factors (see Section 1.2 for more details):

- The cost for station for transmitting within range R is (proportional to) R^α, where $\alpha \geqslant 1$ is a constant depending on the environmental conditions (e.g., in the ideal environment $\alpha = 2$) [18].
- If a station i transmits with range R, then it actually broadcasts to all stations within that range. So, providing a direct connection from i to j also gives a set of connections to other stations "for free": Those stations at distance not larger than $\mathtt{dist}(i, j)$, where $\mathtt{dist}(i, j)$ denotes the distance between i and j.

In order to compute a feasible solution, we need to know the distances from every station i to all other stations. These data are *privately managed* by station i which knows her position with respect to the other ones (this information is typically obtained by exchanging messages with the neighbor stations [18]).

Unfortunately, when a network manager [4] asks such private data (i.e., relative distances) to the stations, the latter may report false values with the hope that the manager assigns them a smaller transmission range. This is where the *selfish* behavior of the node owners plays a crucial role. To avoid this, we aim to design a so called *mechanism*, that is, a set P of payment functions which, combined with a suitable algorithm that constructs a solution (i.e., a range assignment), rewards each station for her expenses in implementing the solution (i.e., the power corresponding to her transmission range). In particular, we want to achieve two goals *simultaneously*:

- Each station maximizes her *utility* (or *profit*) when reporting the true costs, where the utility is computed as payments minus costs; by knowing the mechanism, stations act rationally and report their true distances. A mechanism satisfying this property is called *truthful* (see Section 1.4 for a formal definition).
- The algorithm used in the mechanism (which determines the cost of each station) computes a "good" solution provided that the stations give the correct input (i.e., true distances) to it.

The aim of this paper is to investigate the existence of feasible approximation truthful mechanisms for the above range assignment problem, that is, truthful mechanisms that compute approximate solutions in polynomial-time.

1.1. The model

1.1.1. Range assignments and power consumption

Let S be the set of stations. The range one has to assign to a station i is uniquely determined by the set of connections $E^i \subseteq \{i\} \times S$ that i has to guarantee. Hence, a range assignment can be represented by the set $E = \cup_i E_i$ of all connections that must be maintained in the

[4] Throughout the paper we use the term 'network manager' to denote any protocol computing a range assignment. Thus, a 'network manager' should not be confused with a central authority somehow directly controlling the network resources.

network. The graph $G(S, E)$ is called the *communication graph*. For the sake of brevity, since S is fixed, we will denote $G(S, E)$ simply as E. Clearly the set $\mathcal{O}$ of *feasible* solutions depends on the property we require for E.

The definition of the cost of station i to establish the connections in E^i in the wireless network model is different from that in any wired network model. In the latter, this cost is proportional to the sum of the costs of *all* connections in E^i while, in a wireless network, the connections in E^i can be established by means of a single transmission performed with power sufficient to reach the *farthest* recipient in E^i. The cost of this transmission is determined by the power with which i has to send the message. Let C^i_j be the cost of station i to send a message with (minimal) power sufficient to reach j.

We assume that the range assignment is chosen by a network manager. In order to choose a low-cost solution, the manager needs information about the cost of the connections. So, in the first phase, every station i sends her cost vector $C^i = \langle C^i_1, \ldots, C^i_n \rangle$ to the manager. Let $\mathcal{C} = \langle C^1, \ldots, C^n \rangle$ be the n^2-vector containing all the connection costs. In the second phase, the manager, based on $\mathcal{C}$, computes a suitable range assignment $E \in \mathcal{O}$ and asks station i to transmit messages with power sufficient to maintain E^i.

The cost of station i for maintaining the communication graph E, denoted by $\texttt{cost}^i$, is thus

$$\texttt{cost}^i(C^i, E) := \max_{j\,:\,(i,j)\in E} C^i_j.$$

Hence, the total cost of the range assignment E is

$$\texttt{cost}(\mathcal{C}, E) = \sum_{i\in S} \texttt{cost}^i(C^i, E). \tag{1}$$

Assigning transmission ranges to stations that (i) guarantee a required connectivity property between stations and (ii) minimize the overall power consumption (i.e., the total cost) of the network gives rise to interesting algorithmic questions. In particular, these two aspects yield a class of fundamental optimization problems, denoted as *range assignment* problems, which has been the subject of several recent works in wireless network theory [14,10,6].

In this paper we consider one of the most studied connectivity property: *Strong connectivity* (`SC`). It is thus required that the communication graph induced by the range assignment is strongly connected. This allows all-to-all communication. Optimal solutions of the resulting range assignment problem, denoted as `Min-Range(SC)`, are minimal cost range assignments whose induced communication graph is strongly connected.

1.1.2. Selfishly-acting stations

We consider each station as a selfishly acting agent that *privately knows* part of the input: Station i privately knows C^i that the manager must use for the computation of a feasible solution. To stress that the agent declarations might deviate from the true cost vectors C^i, we shall denote them by D^i.

A *mechanism* for `Min-Range(SC)` is a pair $(\textsc{alg}, P)$, where $\textsc{alg}$ is an algorithm that, on input $\mathcal{D} = \{D^1, \ldots, D^n\}$, returns a feasible range assignment $E = \textsc{alg}(\mathcal{D})$ and $P = P(\mathcal{D}, E)$ is the payment vector. Hence, agents, being selfish, will try to maximize their utility

$$U^i = P^i(\mathcal{D}, E) - \texttt{cost}^i(C^i, E), \quad i = 1, \ldots, n.$$

We require the mechanism to guarantee, for every agent i, the existence of a so called *dominant strategy*, that is, a strategy σ_i such that, for every possible choice of the other agents, agent i maximizes her utility by following strategy σ_i. In this case the mechanism is an *implementation with dominant strategies*. Otherwise, if we only require that the mechanism guarantees the existence of strategies $\sigma_1, \ldots, \sigma_n$ for all agents $1, \ldots, n$ such that any agent i maximize her utility by following σ_i in the case every other agent j follows σ_j (*Nash equilibrium strategy*) we say that the mechanism is a *implementation with Nash strategies*.

Implementations with dominant strategies are thus stronger than those with Nash strategies. A dominant strategy remains the best strategy no matter what the other agents do (in particular, also when some agent does not act rationally). Furthermore, the latter are often impractical from a computational point of view. In fact implementation with Nash strategies require that agents achieve a Nash equilibrium and computing a Nash equilibrium is "a most fundamental computational problem whose complexity is wide open" [19].

A mechanism is *truthful* with respect to dominant strategies (to Nash equilibrium strategies) if declaring the truth (i.e., $D^i = C^i$) is a dominant strategy (a Nash equilibrium).

Though nontruthful implementations with dominant strategies (Nash equilibrium strategies) are possible, Gibbard [11] proved that, if there exists, for a given problem, an implementation with dominant strategies (with Nash equilibrium strategies), then there exists a truthful implementation with dominant strategies (with Nash equilibrium strategies).

For the above reasons we concentrate on mechanisms that are implementations with truthful dominant strategies, from now on simply truthful mechanisms.

1.2. Previous related works

In this section we review the main previous results on assignment problems and some fundamental results on algorithmic mechanism design.

1.2.1. Range assignment problems

The `Min-Range(SC)` problem is known to be NP-hard even when the connection cost vector $\mathcal{C}$ yields an Euclidean 2-dimensional space (e.g., when stations are located on the plane and $\alpha = 1$) [7,14]. On the other hand, `Min-Range(SC)` admits a polynomial-time 2-approximation algorithm [14]. Finally, the problem is polynomially solvable when stations are located on the line (i.e., linear radio networks) [14]. For further results on other versions of the range assignment problem, we refer the reader to [6].

1.2.2. (Algorithmic) Mechanism design

1.2.2.1. Truthful VCG mechanisms. The theory of mechanism design dates back to the seminal papers by Vickrey [21], Clarke [3] and Groves [12]. Their celebrated *VCG mechanism* is still the prominent technique to derive truthful mechanisms for many problems (e.g., shortest path, minimum spanning tree, etc.). In particular, when applied to combinatorial optimization problems (see e.g., [16,20]), the VCG mechanisms guarantee the truthfulness under the hypothesis that the mechanism is able to compute the optimum and the optimization function is *utilitarian*, that is, the optimization function is equal to the sum of the single agents' valuations.

1.2.2.2. Feasible mechanism design. Since for several important optimization problems it is not possible to compute the optimum in polynomial time, unless P = NP, Nisan and Ronen [17] focused on the truthfulness of *VCG-based mechanism*, that is, mechanisms obtained by replacing, in the VCG ones, the exact algorithm with an approximation one.

They first showed that sub-optimal solutions are not suitable because a false declaration may improve the computed solution and therefore the utility of the agent. In particular, they define a class of optimization problems, termed *cost minimization allocation problems* (CMAP) and they proved that truthful VCG-based mechanisms compute, for any CMAP problem, either an optimal solution or a solution arbitrarily far-off the optimum.

To avoid this, they introduce the so called *second chance mechanism*. For CMAP problems, second chance mechanisms only guarantee a weaker form of truthfulness.

1.3. Our contribution

Even though the `Min-Range(SC)` problem is a utilitarian problem (see Eq. (1)), thus admitting a truthful VCG mechanism, such a mechanism requires the computation of a minimum-cost solution. The latter problem is NP-hard [7,14], thus implying that the resulting VCG mechanism cannot run in polynomial time, unless P = NP.

We investigate the existence of *VCG-based* truthful mechanisms. The importance of these mechanisms is twofold: (i) On the one hand, VCG (-based) mechanisms are still the major technique to derive truthful mechanisms; (ii) on the other hand, real problems require fast computations of "sufficiently good" solutions. The latter goal seems to require truthful mechanisms since, in our problem, an optimal algorithm running on false costs may produce arbitrarily bad solutions.

Our first result rules out the possibility of obtaining polynomial-time truthful approximation VCG-based mechanisms. We indeed show that if a VCG-based mechanism (ALG, P) for `Min-Range(SC)` is truthful then it is either optimal or it computes a solution of cost arbitrarily far off the optimum. So, any polynomial-time truthful mechanism for the `Min-Range(SC)` has an *unbounded* approximation ratio unless P = NP. The proof of this negative result can also be derived by showing that the `Min-Range(SC)` is a CMAP (as mentioned in Section 1.2.2, CMAP problems do not admit truthful VCG-mechanism [17]). However, this alternative proof is even longer and counterintuitive.

We then consider the special case in which the true agents costs form a metric space (e.g., stations located on an Euclidean space and $\alpha = 1$). We first observe that our direct proof of the negative result does not apply to metric instances. This led us to investigate this important relevant case. We provide a simple truthful VCG-based mechanism that, when applied to metric instances, returns a constant approximate solution. We emphasize that even this special case is NP-hard [7] and no efficient truthful mechanism was previously known.

The algorithm used in our mechanism was proposed by Calinescu et al. [2]: They showed that this algorithm guarantees a *golden ratio* approximation, that is $(\sqrt{5}+1)/2 \simeq 1.618$. In this paper, we also improve the analysis of that algorithm by achieving a tight approximation ratio of 1.5.

We also prove a similar result for the practically relevant case $1 < \alpha \leqslant 2$ and *well-spread* instances, that is, instances in which stations are located on the plane and any two stations must not be “too close”. Well-spread wireless networks have been studied in [5,8,9]. Because of interference problems, this is the typical situation in radio networks adopted in practice [15,18] (see Section 3.2.2 for a formal definition). Observe that these instances are not metric instances for $\alpha > 1$: Here the agent costs are proportional to the αth power of the relative distances.

Interestingly, our mechanisms are *always* truthful and the approximability does not require the mechanisms to know whether the instance is metric or well-spread. Notice that, even if the true costs yield a metric or a well-spread instance, the declared costs might not satisfy any of these properties. However, since our mechanisms are always truthful, agents are motivated to reveal the truth and so the approximation is guaranteed.

1.3.1. Organization of the paper

In Section 2 we provide the hardness result of the `Min-Range(SC)` problem for the general case. In Section 3 we describe the truthful VCG-based mechanism that, for metric and well-spread instances, guarantees a constant approximation ratio (Section 3.2). Finally, in Section 4 we discuss some open problems.

1.4. Preliminaries

A *mechanism* for `Min-Range(SC)` is a pair (ALG, P), where ALG is an algorithm that, on input $\mathcal{D}$, returns a feasible range assignment $E = \text{ALG}(\mathcal{D})$ and $P = P(\mathcal{D}, E)$ is the payment vector.

Let $\mathcal{D}^{-i}$ denote the vector of declarations of all agents but agent i. Let further $\langle \mathcal{D}^{-i}; C^i \rangle$ denote the vector $\mathcal{D}$ where the declaration of agent i is replaced by C^i. A mechanism (ALG, P) is *truthful* if, for any agent i and for all possible declarations $\mathcal{D}^{-i}$ of the other agents, the corresponding utility

$$U^i = P^i(\mathcal{D}, E) - \mathtt{cost}^i(C^i, E), \tag{2}$$

is maximized when $D^i = C^i$.

Definition 1. A mechanism (ALG, P) is called a *VCG-based mechanism* if the payments

$$P = \langle P^1, \ldots, P^n \rangle$$

are of the form

$$P^i = -\sum_{j \neq i} \mathtt{cost}^j(D^j, E) + h^i(\mathcal{D}^{-i}),$$

where $h^i(\cdot)$ is any function independent of D^i.

Intuitively, these mechanisms achieve truthfulness by relating the utility of an agent with the total cost of the solution chosen by the mechanism: The better the solution the higher the utility. In order to formalize this statement let us consider the utility of agent i. From (2) we get

$$\begin{aligned} U^i &= P^i - \mathtt{cost}^i(C^i, E) \\ &= h^i(\mathcal{D}^{-i}) - \mathtt{cost}^i(C^i, E) - \sum_{j \neq i} \mathtt{cost}^j(D^j, E) \\ &= h^i(\mathcal{D}^{-i}) - \mathtt{cost}(\langle \mathcal{D}^{-i}; C^i \rangle, E). \end{aligned}$$

Since $h^i(\mathcal{D}^{-i})$ is independent of her declarations, agent i tries to minimize $\mathtt{cost}(\langle \mathcal{D}^{-i}; C^i \rangle, E)$. Hence, her declaration should be chosen in order to let the algorithm return a solution $\tilde{E}$ which minimizes $\mathtt{cost}(\langle \mathcal{D}^{-i}; C^i \rangle, E)$. This can be achieved by declaring the truth, assuming that the mechanism, by using ALG, is able to find an optimal solution $\tilde{E}$.

Definition 2. A mechanism (ALG, P) is a *VCG mechanism* if it is a VCG-based mechanism and

$$\forall \mathcal{D} \quad \text{ALG}(\mathcal{D}) \in \arg\min_{E \in \mathcal{O}}(\mathtt{cost}(\mathcal{D}, E)),$$

where $\mathtt{cost}(\mathcal{D}, E) = \sum_{i=1}^{n} \mathtt{cost}^i(D^i, E)$ and $\mathcal{O}$ is the set of all possible outputs.

Theorem 3 (*Groves [12]*). *VCG mechanisms are truthful*.

Finally, let $E^* \in \arg\min_{E \in \mathcal{O}}(\mathtt{cost}(\mathcal{D}, E))$, we define $\mathtt{opt}(\mathcal{D}) = \mathtt{cost}(\mathcal{D}, E^*)$.

2. VCG-based mechanisms for `Min-Range(SC)`: The hardness result

As mentioned in the Introduction, finding an optimal solution in polynomial time for the `Min-Range(SC)` problem is NP-hard. So, it turns out that efficient mechanisms should make use of approximating solutions. However, we next show that any reasonable truthful VCG-based mechanism (see Definition 1) requires an algorithm that computes an optimal solution.

A mechanism (ALG, P) is called *degenerate* if the approximation ratio produced by ALG is unbounded, i.e., for any $R > 0$, there exists a true cost vector $\mathcal{C}$ such that

$$\frac{\mathtt{cost}(\mathcal{C}, \text{ALG}(\mathcal{C}))}{\mathtt{opt}(\mathcal{C})} \geqslant R.$$

Theorem 4. *If a VCG-based mechanism* (ALG, P) *for* `Min-Range(SC)` *is truthful then it is either optimal or degenerate*.

Proof. Let (ALG, P) be a non-optimal truthful VCG-based mechanism for `Min-Range(SC)`. We will show that it is degenerate. Since ALG is not optimal, there exists a cost vector $\mathcal{C}$ and a communication graph Y (i.e., the optimal communication graph) for which $\mathtt{cost}(\mathcal{C}, \text{ALG}(\mathcal{C})) > \mathtt{cost}(\mathcal{C}, \mathrm{Y})$.

Let us define $\hat{C} = \langle \hat{C}^1, \ldots, \hat{C}^n \rangle$ where, for any $i, j = 1, \ldots, n$,

$$\hat{C}^i_j = \begin{cases} C^i_j & \text{if } (i, j) \in \mathrm{Y}, \\ \alpha & \text{otherwise}, \end{cases} \tag{3}$$

where α is any positive "large" constant such that $\alpha > \max\{C^i_j \mid i, j = 1, \ldots, n\}$. Now, we construct the following sequence of cost vectors:

$$\mathcal{S}_0 = \mathcal{C}, \quad \mathcal{S}_1 = \langle \hat{C}^1, C^2, \ldots, C^n \rangle, \ \ldots, \ \mathcal{S}_n = \langle \hat{C}^1, \ldots, \hat{C}^n \rangle.$$

Claim 5. *For any* $z = 0, \ldots, n$, *it holds that* $\texttt{cost}(\mathcal{S}_0, \mathrm{Y}) = \texttt{cost}(\mathcal{S}_z, \mathrm{Y})$.

Proof. Notice that in $\mathcal{S}_z$, only the values of edges which are not in Y change. Since those do not influence the value of $\texttt{cost}(\mathcal{S}_z, \mathrm{Y})$, the claim follows. □

The next claim states that the mechanism fails to find the optimal solution Y also for the cost vector $\mathcal{S}_n$.

Claim 6. $\texttt{cost}(\mathcal{S}_n, \mathrm{ALG}(\mathcal{S}_n)) > \texttt{cost}(\mathcal{S}_n, \mathrm{Y})$.

Proof. Because ALG is not optimal on $\mathcal{C} = \mathcal{S}_0$, we have

$$\texttt{cost}(\mathcal{S}_0, \mathrm{ALG}(\mathcal{S}_0)) > \texttt{cost}(\mathcal{S}_0, \mathrm{Y}).$$

We next prove that, for all $z = 1, \ldots, n$

$$\begin{aligned} \texttt{cost}(\mathcal{S}_z, \mathrm{ALG}(\mathcal{S}_z)) &\geqslant \texttt{cost}(\mathcal{S}_{z-1}, \mathrm{ALG}(\mathcal{S}_z)) \\ &\geqslant \texttt{cost}(\mathcal{S}_{z-1}, \mathrm{ALG}(\mathcal{S}_{z-1})). \end{aligned}$$

The second inequality holds because the mechanism is assumed to be truthful. Indeed, if it would not hold, agent z would be better off declaring $\hat{C}^z$ instead of C^z. The first inequality holds since, by the choice of α, it holds that $\mathcal{S}_{z-1} \leqslant \mathcal{S}_z$ component wise. By combining all the $2n + 1$ inequalities with Claim 5, we get the thesis. □

Let us now consider the cost vector $\mathcal{S}_n$ and the communication graph $\mathrm{X} = \mathrm{ALG}(\mathcal{S}_n)$. From Claim 6, it follows that X is not optimal and an edge $e \in \mathrm{X}$ exists which is not in Y. Indeed, from Eq. (1), if $\mathrm{X} \subseteq \mathrm{Y}$ then it would hold that $\texttt{cost}(\mathcal{D}, \mathrm{X}) \leqslant \texttt{cost}(\mathcal{D}, \mathrm{Y})$ for any $\mathcal{D}$.

Let i be the agent (node) to which e belongs to. Since $\hat{C} = \mathcal{S}_n$, the cost of agent i in the solution X is $\texttt{cost}^i(\hat{C}^i, \mathrm{X}) = \alpha$. Hence, $\texttt{cost}(\mathcal{S}_n, \mathrm{ALG}(\mathcal{S}_n)) \geqslant \alpha$. From Claim 5, we know that $\texttt{cost}(\mathcal{S}_n, \mathrm{Y})$ does not depend on α, thus implying that

$$\frac{\texttt{cost}(\mathcal{S}_n, \mathrm{ALG}(\mathcal{S}_n))}{\texttt{cost}(\mathcal{S}_n, \mathrm{Y})}$$

is unbounded, so the mechanism is degenerate. □

3. A truthful mechanism for metric and well-spread instances

In this section, we derive a mechanism for the `Min-Range(SC)` problem which is always truthful and it achieves a constant approximation for both metric (i.e., $\alpha = 1$) and well spread-instances. The metric version of the problem will be denoted as `Metric-Min-Range(SC)`.

The `Hub` *algorithm*: The algorithm adopted by our mechanism is simple: It computes a range assignment that contains a directed minimum spanning tree (of the complete directed graph induced by the agent declarations) oriented towards a sink node s and all the outgoing edges from s. Calinescu et al. [2] proved that this *Hub* algorithm guarantees a performance ratio of $(\sqrt{5}+1)/2 \simeq 1.618$ for the metric case.

We say that a graph is an *hub-tree* if it contains a directed spanning tree towards a sink node s and all outgoing edges from s (Fig. 1).

3.1. A truthful VCG-based mechanism

Let us consider the VCG-based mechanism $\mathcal{M} = (\texttt{Hub}, P)$ where `Hub` is the algorithm defined above and P is defined as

$$P^i = -\sum_{j \neq i} \texttt{cost}^j(D^j, \texttt{Hub}(\mathcal{D})).$$

Theorem 7. *The mechanism* $\mathcal{M} = (\texttt{Hub}, P)$ *is a truthful mechanism for the* `Min-Range(SC)` *problem.*

Proof. Proposition 3.1 in [17] states that a VCG-based mechanism with an output algorithm that is maximal in its range is truthful. Hence the theorem simply follows by observing that the `Hub` algorithm computes a *hub-tree* of minimal cost. □

3.2. The approximation property of the algorithm

In the sequel, thanks to Theorem 7, we will assume that the declared costs induce a Euclidean instance.

```
INPUT: The declared agent costs D.
OUTPUT: A communication graph E representing a feasible
        range assignment.
begin
    forall c = 1, ..., n do
        T(c) ← Minimum spanning tree directed towards c
               according to D;
        E(c) ← T(c) ∪ {(c, j) s.t. j ≠ c}
    return E ← argmin_{E(c):1≤c≤n} (cost(D, E(c)))
end
```

Fig. 1. The `Hub` algorithm.

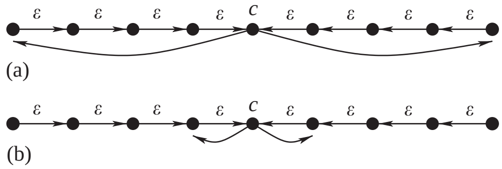

Fig. 2. Worst case for the Hub algorithm.

3.2.1. The metric case

Theorem 8. *The* Hub *algorithm guarantees a* $3/2$ *approximation factor for the* Metric-Min-Range(SC) *problem.*

It is easy to verify that this bound is tight for the Hub algorithm. Indeed, consider the instance in Fig. 2: The cost of the optimal solution is $\texttt{cost}(\mathcal{C}, T) + \varepsilon$ (Fig. 2b) whereas the solution of the Hub algorithm has a cost $\frac{3}{2}\texttt{cost}(\mathcal{C}, T)$ (Fig. 2a).

The proof of Theorem 8 needs some preliminary technical results.

Lemma 9. *Let* $\mathcal{C}$ *be the agent cost vectors and let* w_{M} *be the maximum edge-cost of the minimum spanning tree* T. *Then it holds that*

$$\texttt{opt}(C) \geqslant \texttt{cost}(C, T) + w_{\mathrm{M}}.$$

Proof. Let E^* be an optimal solution for Metric-Min-Range(SC) and let v be any station in S. As observed in the analysis of the 2-approximation algorithm in [14], E^* must contain a minimum spanning tree T directed towards v. Then

$$\texttt{opt}(C) \geqslant \texttt{cost}(\mathcal{C}, T) + \texttt{cost}(C^v, (E^*)^v).$$

Since there exists (at least one) v such that $\texttt{cost}(C^v, (E^*)^v) \geqslant w_{\mathrm{M}}$, then the thesis holds. □

Definition 10. The *eccentricity* of a node v in a connected weighted graph G is the weight of the maximum among all the shortest paths between v and every other point in G. The minimum graph eccentricity is called the *graph radius*. A point v is a *central point* of a graph if the eccentricity of the point equals the graph radius. The set of all central points is called the *graph center* or, simply, *center*.

The proof of the following result directly follows from Definition 10 and from the Hub algorithm.

Fact 11. *The sink agent chosen by the* Hub *algorithm is a central point of the minimum undirected spanning tree induced by the agent true costs* $\mathcal{C}$.

The following is a classical result in graph theory.

Theorem 12 (*Jordan [13]*). *In a weighted tree T, with positive weights, the graph center contains one node or two adjacent nodes.*

Proof. The proof follows by observing that the subgraph induced by the graph center of a weighted graph G is always a clique. This easily implies that the graph center of a tree contains no more than two vertices. □

A weighted tree is thus denoted as *central tree* when its center is a single node and as *bi-central tree* otherwise.

Lemma 13. *Let T be a weighted tree with positive weight function $w : T \to \mathbb{R}^+$. If R is the radius of T and $w_{\mathrm{M}} = \max_{e\in T} w(e)$* then

$$R \leqslant \frac{w(T) + w_{\mathrm{M}}}{2}$$

where $w(T) = \sum_{e\in T} w(e)$.

Proof. Let us first assume that T is a bi-central tree (see Theorem 12) whose central points are x and y connected by an edge of weight $w(x, y)$. It is easy to verify that the two paths P_x and P_y, that testify the eccentricity of x and y respectively, share only the edge (x, y), that is

$$P_x \cap P_y = (x, y).$$

Indeed, this intersection is not-empty (otherwise the eccentricity of x and y would be $w(P_x) + w(x, y) = w(P_y) + w(x, y) > R$) and does not contain other edges (because T is a tree). It thus follows that

$$w(T) \geqslant w(P_x) + w(P_y) - w(x, y) \geqslant 2R - w_{\mathrm{M}}.$$

Hence, the thesis holds for bi-central trees.

As for central trees, let c be the central point of T and P_c be the path from c that testifies the eccentricity of c. We claim that there exists (at least) one path P'_c which starts from c, it is disjoint from P_c, and such that

$$w(P'_c) + w(c, x) > w(P_c),$$

where x belongs to P_c. Indeed, let P_x be the path starting from x whose length testifies the eccentricity of x. Since c is the only central point then $w(P_x) > w(P_c)$. Moreover, P_x passes through c because, otherwise, $w(P_x) \leqslant w(P_c) - w < w(P_c)$ (see also Fig. 3). It then holds that

$$w(T) \geqslant w(P_c) + w(P_x) - w(c, x) > 2R - w_{\mathrm{M}}$$

and the thesis follows. □

Proof of Theorem 8. Given any "metric" cost vector $\mathcal{C}$ the total cost of $\mathtt{Hub}(\mathcal{C})$ satisfies

$$\mathtt{cost}(\mathcal{C}, \mathtt{Hub}(\mathcal{C})) = \mathtt{cost}(\mathcal{C}, T) + R.$$

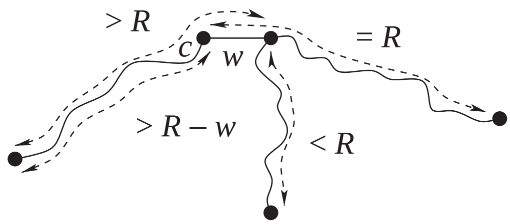

Fig. 3. Central tree.

From Fact 11, R is the cost of the sink station then, by combining Lemma 9 and Lemma 13, the thesis follows. $\square$

3.2.2. Well-spread instances

Let us consider the case in which the stations are located on the 2-dimensional Euclidean space and, for any $i, j = 1, \dots, n$, it holds that $C_j^i = \mathtt{dist}(i, j)^2$, where $\mathtt{dist}(i, j)$ is the Euclidean distance. This case corresponds to that in which stations lie in the empty space. A family $\mathcal{S}$ of instances is *well-spread* if a positive constant γ exists such that, for any $S \in \mathcal{S}$,

$$\mathtt{diam}(S) \leqslant \frac{\delta(S)\sqrt{n}}{\gamma},$$

where $\delta(S) = \min\{\mathtt{dist}(i, j) \mid i \neq j\}$ and $\mathtt{diam}(S) = \max\{\mathtt{dist}(i, j) \mid i, j = 1, \dots, n\}$. Informally speaking, in a well-spread instance, any two stations must be not "too close". Because of interference problems, this is the typical situation in radio networks adopted in practice [15,18].

Theorem 14. *The* `Hub` *algorithm guarantees a* $\mathrm{O}(\gamma)$ *approximation ratio for the* `Min-Range(SC)` *problem on well-spread instances.*

Proof. We have that the performance ratio of the `Hub` algorithm satisfies the following bound

$$\frac{\mathtt{cost}(\mathcal{C}, \mathtt{Hub}(\mathcal{C}))}{\mathtt{opt}(\mathcal{C})} \leqslant \frac{w(T) + \mathtt{diam}(S)^2}{w(T)},$$

where $w(T)$ denotes the cost of a minimum spanning tree for $\mathcal{C}$. Since $w(T) \geqslant (n-1)\delta(S)^2$ and $\delta(S) \geqslant \gamma\, \mathtt{diam}(S)/\sqrt{n}$, we get

$$\frac{\mathtt{cost}(\mathcal{C}, \mathtt{Hub}(\mathcal{C}))}{\mathtt{opt}(\mathcal{C})} = \mathrm{O}(\gamma). \qquad \square$$

4. Conclusions and open problems

In this paper we investigated the existence of truthful VCG-based mechanism for the `Min-Range(SC)` problem. We proved that, in general, every polynomial-time truthful

VCG-based mechanism computes a solution of cost far-off the optimum (unless P=NP) and that there exists a polynomial-time truthful VCG-based mechanism achieving constant approximation for metric and well-spread instances.

Several interesting problems related to our results are still open. The most important, in our opinion, are those listed below.

- The existence of VCG-based mechanisms for the `Min-Range(SC)` problem restricted to the case $\alpha = 2$.
- No result is known concerning mechanism design for the `Min-Range(`Π`)` problem when the property Π is different from the strong connectivity. We remark that for many of the these problems the negative result of Theorem 4 holds. When the required connectivity property is a directed spanning tree from a source station (i.e., the broadcast property) the problem is NP-hard for $\alpha > 1$ [4] and it is approximable within a constant [1,4]. It is thus interesting to investigate whether it is possible to derive an approximation VCG-based mechanism exploiting the results in [1,4].
- In our model the private information of each station is the set of distances with respect to all other stations. Other reasonable private information can be considered: For instance we can assume that the protocol has a partial knowledge of the network topology.
- Even though the VCG method is the major technique to obtain truthful mechanism, a fundamental future research is to develop alternative methods to manage selfish behavior in the context of energy consumption in wireless networks.

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