Equilibria for Broadcast Range Assignment Games in Ad-Hoc Networks

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Abstract. Ad-hoc networks are an emerging networking technology, in which the nodes form a network with no fixed infrastructure: each node forwards messages to the others by using the wireless links induced by their power levels. Generally, energy-efficient protocols heavily rely on cooperation. In this paper, we analyze from a game-theoretic point of view the problem of performing a broadcast operation from a given station s. We show both theoretical and experimental results on how the existence of (good) Nash equilibria is determined by factors such as the transmission power of the stations or the payment policy that stations can use to enforce their reciprocal cooperation.

1 Introduction

Ad-hoc networks do not need any fixed infrastructure for communication: nodes consist of radio stations that are able to communicate by sending messages with a certain power. This feature is particularly attractive for users since they do not have to rely on a service provider for building/using the network.

Tipically, stations are located in a two-dimensional Euclidean space and are connected by *wireless links* that are induced by their power levels. Each station v is equipped with an *omnidirectional antenna* and, depending on the environmental conditions, a signal transmitted with power P_v can be received by every other station t such that

$$d(v,t)^\alpha \leq \frac{P_v}{\gamma}, \tag{1}$$

⋆ Supported by the European Union under the Project IST-2001-33135 "Critical Resource Sharing for Cooperation in Complex Systems" (CRESCCO).

⋆⋆ Partially supported by the Italian Research Project PRIN 2003 "Optimization, simulation and complexity of the design and management of communication networks"

V.R. Sirotiuk and E. Chávez (Eds.): ADHOC-NOW 2005, LNCS 3738, pp. 4–17, 2005.

where $d(v,t)$ is the Euclidean distance between v and t, $\alpha \geq 1$ is the distance-power gradient, and $\gamma \geq 1$ is the transmission quality parameter. In an ideal environment (i.e., in empty space) it holds that $\alpha = 2$, but it may vary from 1 to more than 6 depending on the environment conditions at the location of the network (see [16]). According to the previous equation, when a station v transmits with power P_v, it covers an area consisting of all points at distance at most $r_v \geq (P_v/\gamma)^{1/\alpha}$ from v. The value r_v is the *transmission range* of v, i.e., the maximum distance at which station v can transmit in one hop with power P_v. Hence, assigning transmission ranges to the stations is equivalent to decide their transmission powers. In the remaining of this work, we assume $\gamma = 1$, although all of our results easily apply to any constant γ.

The set of all transmission ranges yields a *range assignment* that is a function $r : S \to \mathbb{R}^+$, where S denotes the set of stations and $r(v) = r_v$. We consider *broadcast range assignments*, that is, range assignments which, given a source station $s \in S$, allow this station to transmit to all other stations (via a multi-hop communication). Formally, consider a *transmission graph* $G_r = (S, E_r)$, such that $(v,t) \in E_r$ if and only if $d(v,t) \leq r(v)$. Then r is a broadcast range assignment if G_r contains a directed spanning tree rooted at s.

The *social cost* (or, simply, the cost) of a (broadcast) range assignment is measured as the overall energy that all stations in the network spend to implement these ranges, that is,

$$\texttt{cost}(r) = \sum_{v \in S} r(v)^\alpha.$$

If ranges are assigned to stations by a central authority, then it is possible to get broadcast range assignments whose cost do not differ to much from the optimum cost (see Subsection 1.2). Implicit in this approach is the assumption that each station will actually transmit with the range specified by the authority. This assumption cannot be take for granted in a (more realistic) scenario in which stations are managed by different (potentially selfish) users. This is indeed the case of ad-hoc networks for which it is fundamental to develop mechanisms that enforce stations cooperation.

In this work, we consider a game-theoretic setting in which each station corresponds to a different player (or agent) of a game named *broadcast range assignment game*. The strategy of each player v is to decide its transmission range $r(v)$ and/or to provide some payment to some other players in order to convince them to transmit with a given range.

The range assignment r derived by the strategies of all players can induce a *benefit* $b_v(r)$ to every station v. The benefit can represent, for example, the interest of station v in guaranteeing the given connectivity, the sum of the payments received/provided from/to the other stations, or a combination of these two things. Since implementing the range $r(v)$ induces a cost of $r(v)^\alpha$, we can define a *utility function*

$$\mu_v(r) = b_v(r) - r(v)^\alpha \tag{2}$$

that station v aims at maximizing. Observe that $\mu_v(r)$ depends on the strategies of *all* the stations. In particular, if station v' changes its transmission range from $r(v')$ to $r'(v')$, then we obtain a range assignment r' and the utility of all stations v will change to $\mu_v(r')$.

We are interested in sets of "stable" strategies for which no player has an incentive in unilaterally switching to a different strategy. These configurations are known as Nash equilibria [15]. A range assignment r is a *Nash equilibrium* if $\mu_v(r) \geq \mu_v(r')$, for every station v and for every r' obtained from r by changing $r(v)$ into $r'(v)$. We are interested in *good* Nash equilibria, that is, strategies that minimize the overall power consumption (the social cost). Sometimes, it can be convenient to consider ϵ-*approximate* Nash equilibria, that is, range assignments r that guarantee $\epsilon \cdot \mu_v(r) \geq \mu_v(r')$, for all v and r'.

As already noticed, some station v may be interested in guaranteeing the given connectivity requirement (i.e. broadcast) and may be willing to pay some other station in order to maintaining the needed connectivity property. We model the connectivity requirement of a station v by saying that v is *penalized* if its connectivity requirement is not satisfied by the range assignment r. In this case, we define $b_v(r) = -\infty$ and thus $u_v(r) = -\infty$ as well (see Eq. 2). Otherwise, we define $b_v(r)$ as the "balance" derived from all money exchanged with the other stations that is,

$$b_v(r) = \sum_{u \in S} \left(p_u^v(r) - p_v^u(r)\right),$$

where $p_u^v(r)$ is the payment from station u to station v when range assignment r is implemented.

The simplest games we consider are the *Payments-free* games. Here, no payments are allowed (as in [9]). Clearly, a broadcast range assignment will be a Nash equilibrium if at least one station is penalized.

We next consider payment games in which a pricing policy is defined that will depend on which transmission ranges are "used" by a station v, given a range assignment r. In particular, payments provided by a station v are used to allow this station to increase the transmission range of other stations in order to create a path from s to v. In this case, the strategy of node v consists in specifying a path from s to v. Our payment policies differ with respect to which stations in the chosen path will receive a payment from v. For every station u, we define $\texttt{Used}_r(u)$ as the set of stations having to pay u.

- *Edge-payment.* Here we consider a simple local policy in which payments associated to the range assignment r are only provided to neighbor stations. In particular, assume u is the last station in the path from s to v. Then, station u is the only station that receives a payment from v. For every such pair of stations u and v, $v \in \texttt{Used}_r(u)$.
- *Path-payment.* In this case, station v can be required to pay for (some part of) all the ranges $r(s), r(u_1), \ldots r(u_k)$, where $\langle s = u_0, u_1, \ldots, u_k, v\rangle$ is the strategy of v. In this case, $v \in \texttt{Used}_r(u_i)$ for all $i = 0, \ldots, k$.

We then consider two possible payment policies:

- *No-profit.* The cost $r(u)^\alpha$ is divided among all stations $v \in \texttt{Used}_r(u)$ (similarly to [2,3]).
- *Profit.* Every station v using station u pays exactly $p_v^u(r) = r(u)^\alpha$. Clearly, station u may have a profit if $|\texttt{Used}_r(u)| > 1$.

The connectivity requirements we consider are of two types: (i) *Reachability*, that is, station v has $b_v(r) = -\infty$ whenever r does not allow s to transmit to v, and (ii) *B-Broadcasting*, that is, for all $v \in B \subseteq S$, $b_v(r) = -\infty$ whenever r is not a broadcast range assignment. In particular, we consider s-Broadcasting and S-Broadcasting, i.e., only the source s or all stations in S are interested in the information dissemination, respectively.

1.1 Paper Contribution

We investigate the existence of Nash equilibria, the computational complexity of finding (a good) one, and convergence properties to Nash equilibria under the natural *best response*[1] assumption. For the latter, we take into account both the convergence time and the quality of the final Nash equilibrium. A significant measure of the quality of a Nash equilibrium is the *price of stability*, that is, the ratio between the cost of the equilibrium and the cost of the optimal solution (that, in general, is not a Nash equilibrium).

We also consider a weaker notion of ϵ-approximate Nash equilibria since we observe that, for some of the games we consider, ϵ-approximate Nash equilibria are difficult to obtain, even when agents changing their strategies can only attain a very small gain. We thus introduce the concept of *Payment ϵ-approximate Nash equilibrium*, which takes into account these aspects.

Regarding the Payment-free games, we prove that the s-Broadcasting, the s-Broadcasting with Reachability and the S-Broadcasting have Nash equilibria. In particular, the *unique* equilibrium for the s-Broadcasting and for the s-Broadcasting with Reachability can be arbitrary more expensive than the optimal solution, whereas *all* the Nash equilibria for the S-Broadcasting game are optimal range assignments (NP-hard to be computed).

Table 1 summarizes our main results for the Reachability problem in the profit models for the Edge- and Path-payment policies.

Finally, we experimentally evaluate the behavior of an algorithm that looks for a Nash equilibrium for the Reachability game in the No-profit model for both the payment policies we have introduced (Edge- and Path-payment). We test this algorithm on thousands of random instances and instances derived by the mobility model described in [11]. We obtain the following results: (i) the algorithm converges to a Nash equilibrium for all the generated instances; (ii) the convergence of the algorithm is guaranteed in a bounded number of steps that weakly depends on the size of the instances; (iii) the Nash equilibrium

[1] Best response strategies assume each player to select the strategy that currently maximize its utility [15].

Table 1. Results for the Reachability problem in the profit models for the Edge- and Path-payments policies

Payment	Profit	No-Profit
Edge-	A polynomial time computable Nash equilibrium that is a 6 approximation of the optimal solution	Experimental evaluation + A polynomial time computable Payments 6-approximated Nash equilibrium that is a 6 approximation of the optimal solution
Path-	A polynomial time computable Payments ϵ-approximated Nash equilibrium that is a $6(1+\frac{2}{1-\epsilon})$ approximation of the optimal solution	Experimental evaluation

created by the algorithm is a constant approximation of the optimal solution and (iv) the intermediate configurations are feasible solutions whose cost is an approximation of the optimal cost.

1.2 Related Works

(Broadcast) Range Assignment. The broadcast range assignment problem has been deeply investigated from the point of view of centralized/distributed algorithms. In both cases, the underlying assumption is that stations will always implement the solution computed by such algorithms, even if this solution will not be advantageous for themselves. Several heuristics for this problem version have been proposed [10,12,17]. Among those, the MST-based[2] algorithm has been proved to achieve, for $\alpha \geq 2$, a *constant* approximation ratio [5,7]. A tight bound of 6 has been achieved in [1] (the lower bound of 6 is due to [5]). Interestingly, the (analysis of the) MST-based algorithm turns out to be useful for studying other heuristical approaches: indeed, several of these can be proved to produce a cost which is bounded from above by the cost of MST-based solutions [5], or to be only a constant factor away from the latter [17]. No approximation algorithm is known for $1 < \alpha < 2$. The problem is known to be NP-hard for all $\alpha > 1$ [6], while the case $\alpha = 1$ is trivially in P.

Nash Equilibria and Network Design Games. In [2] the authors introduce *network design games*: each agent offers to pay for an *arbitrary* fraction of the cost of building/maintaining a link of a network, and the corresponding link “exists” if and only if enough money is collected from all agents. Also the agents have a *connectivity requirement* and Nash equilibria correspond to those strategies for which no agent can reduce its payments still having its connectivity fulfilled. In this game, (pure) Nash equilibria may not exist for point-to-point connectivity requirements [2]. (Notice that mixed – i.e., randomized – strategies are meaningless for these games due to the fact that $u_v = -\infty$ if the graph does not support v's connectivity requirement.)

[2] This algorithm is denoted as BLiMST algorithm in [10].

Fixing a "fair" pricing policy in which the cost of an edge is evenly divided among all agents using it ensures the existence of pure Nash equilibria [3]. The result is an application of *potential functions* [13], which the authors use to bound the *price of stability* – i.e., the loss of performance due to this "strict" pricing policy ensuring Nash equilibria. Indeed, given k agents, the *best* Nash equilibria attains a cost of at most $O(\log k)$ the optimum [3]. For directed graphs, this bound is tight [3].

Network design games in ad-hoc wireless networks have been first considered in [9] for point-to-point and strong connectivity requirements. In this game every station has to choose its own transmission range. For point-to-point connectivity, the problem admits pure Nash equilibria and there exists an algorithm to find one of them of cost at most twice the optimum [9]. Conversely, strong connectivity games do not always have Nash equilibria, and not even ϵ-approximate Nash equilibria, for any $\epsilon > 1$. In [4], the authors deal with the multicast games in general ad-hoc networks introducing a pricing policy similar to the one introduced in [3] and they prove that the games induced by these payments have a Nash equilibrium but, finding such equilibrium is NP-hard.

2 Analytic Results

Due to the lack of space, the proofs of the results in this section will be given in the full version of the paper (see [8] for a preliminary draft).

2.1 Payments-Free Games

In the payments-free games messages are forwarded for free. This means that only stations that are penalized when broadcast cannot occur have positive ranges. Hence, broadcast is not supported in the model in which only the stations that do not receive the broadcasted message are penalized.

Proposition 1. *For the s-Broadcasting and the s-Broadcasting with Reachability games (i.e. s and the non-receiving stations are penalized) the only Nash equilibrium is the range assignment in which $r(s) = \max\{d(s, v) : v \in S - \{s\}\}$ and $r(v) = 0$ for any $v \in S - \{s\}$.*

This result implies that the cost-quality ratio is unbounded.

Proposition 2. *For the S-Broadcasting game the only Nash equilibria are the minimum cost broadcast range assignments.*

Proposition 3. *Consider an s-Broadcasting game in which s could pay other stations v an amount $p_s^v = r(v)^\alpha$ for implementing a certain range $r(v)$. Then, only minimum cost broadcast range assignments are Nash equilibria for this game.*

By comparing Proposition 1 and 3 we observe that the introduction of payments may reduce the cost-quality ratio while, on the other hand, it makes the computation of Nash equilibria to become NP-hard [7].

2.2 Payments-Games for Reachability Games

Profit Models: Both Edge- and Path-payment models do admit Nash equilibria which can be found in polynomial time.

Proposition 4. *There exists an Edge-payment policy based on the profit models such that any range assignment yielded by a minimum cost spanning tree of the complete Euclidean graph G derived from the instance is a Nash equilibrium.*

From [1] we can obtain the following result:

Theorem 1. *The cost-quality ratio of the Reachability games under the Edge-payment policy is* 6*.*

Similarly to Proposition 4 we can prove the following:

Proposition 5. *There exists a Path-payment policy based on the profit models such that any range assignment yielded by a shortest path tree rooted at s of the complete Euclidean graph G derived from the instance is a Nash equilibrium.*

Unfortunately, the shortest path tree does not guarantee any approximation of the optimal solution. Moreover, even ϵ-approximate Nash equilibria are difficult to obtain since in the utility function

$$\mu_v(r) = \sum_{u \in S} \left(p_u^v(r) - p_v^u(r) \right) - r(v)^\alpha$$

station v can only affect the payments it provides to the others, while the money received depends only on the other stations' strategies. This means that a considerable change in the station strategy may result in a negligible change in the station utility. However, if we limit our requirements to some weaker notion of approximate equilibria, then, for the Path-payment games the social optimum can be approximated by such equilibria. In the following we define the notion of *payments ϵ-approximate Nash equilibrium.*

Definition 1 (Payments ϵ-approximate Nash equilibria). *A range assignment r is a* Payments ϵ-approximate Nash equilibrium *if, for any station v, and any range assignment r' derived from r by changing only v's strategy, it holds that $\sum_{u \in S} p_v^u(r) \leq \epsilon \sum_{u \in S} p_v^u(r')$.*

Remark 1. Let r be a Payments ϵ-approximate Nash equilibrium. Then, r is an ϵ-approximate Nash equilibrium for the game in which (i) station v cannot refuse to implement a transmission range $r(v)$ if receiving an amount of money not smaller than $r(v)^\alpha$, (ii) a station strategy is to choose a path (thus providing the corresponding money) for being reached, and (iii) the utility of station v is the inverse of the sum of all payments provided to the other agents (or $-\infty$ if not reached).

Theorem 2. *For the Reachability Path-payment game it is possible to compute in polynomial time a Payments ϵ-approximate Nash equilibrium r such that* $\mathtt{cost}(r) \leq 6\,(1 + 2/(1-\epsilon))) \cdot OPT$*, where OPT is the optimum social cost, for any $\epsilon > 1$.*

As a final remark, notice that the profit models introduced in this section require some form of encryption. Actually, a station transmitting with some range r reaches *all* stations at distance r while only those stations having paid their fee must be reached.

No-Profit Models. We consider both the Edge-payment model and the Path-payment model.

We now define our specific Edge-payment policy. Suppose stations $u_1, \ldots, u_k$ receive from station v and suppose that $d(v, u_1) \leq \ldots \leq d(v, u_k)$, that is, station v transmits at range $d(v, u_k)$. Let $r_1 < \ldots < r_h$ be the set of distinct distances between v and any station $u_1, \ldots, u_k$ $(h \leq k)$. Let $N_v(r_j)$ be the set of stations in $\{u_1, \ldots, u_k\}$ at distance exactly r_j from v.

The payments in the Edge-payment model are defined as follows:

$$p_u^v(r) = \sum_{i=1}^{j:r_j=d(v,u)} \frac{r_i^\alpha - r_{i-1}^\alpha}{|N_v(r_i)|}$$

Intuitively speaking, each increment $(r_i^\alpha - r_{i-1}^\alpha)$ in the transmission power of v is equally shared among all the stations using the new range (r_i). Hence, the total amounts of payments received by v equals the energy spent by v for implementing the range $r(v)$.

The Edge-payment model guarantees the existence of an easy to compute Payment ϵ-approximate Nash equilibrium, as stated in the next theorem.

Theorem 3. *Let T be a minimum spanning tree of the complete geometric graph induced by S and let Δ be the maximum out-degree of T. Then it is possible to compute in polynomial time a Payments $(\Delta + 1)$-approximate Nash equilibrium in the Edge-payment model game.*

Since every geometric spanning tree T can be transformed in polynomial time into a spanning tree T' such that T' has the same cost of T and every node in T' has at most 5 neighbors [14], we can conclude with the following result.

Corollary 1. *For the Edge-payment model game it is possible to compute in polynomial time a Payments* 6*-approximate Nash equilibrium.*

We now define our specific Path-payment model. Suppose stations $u_1, \ldots, u_k$ receive from v and suppose that $d(v, u_1) \leq \ldots \leq d(v, u_k)$, that is, station v transmits at range $d(v, u_k)$. Let $r_1 < \ldots < r_h$ be the set of distances between v and any station $u_1, \ldots, u_k$ $(h \leq k)$. Let T be the directed tree rooted at s induced by the range assignment r and T_v be the subtree of T rooted at v.

Define $T_v(r_j)$ as the tree obtained by T_v by removing all the subtrees T_{u_i} such that $d(v, u_i) \neq r_j$.

Let $P_v = \{v_0 \equiv s, v_1, \ldots, v_\ell \equiv u\}$ be the path in T from s to u, then for $i = 1, \ldots, \ell$

$$p_u^{v_i}(r) = \sum_{h=1}^{j:r_j = d(v_h, v_{h-1})} \frac{r_h^\alpha - r_{h-1}^\alpha}{|T_{v_i}(r_h) - \{v_i\}|}$$

Intuitively speaking, each increment $(r_h^\alpha - r_{h-1}^\alpha)$ in the transmission power of v_i is equally shared among all the stations using the new range (r_j) in their paths. Hence, the total amounts of payments received by v_i equals the energy spent by v_i for implementing the range $r(v_i)$.

These payments are introduced in [4]. The authors of this paper prove that this kind of payments always induce a Nash equilibrium, however computing such equilibrium is NP-hard. Moreover, from the analysis in [3], it is possible to derive a upper bound of the cost-quality ratio that is logarithmic on the number of stations. This upper bound is not necessarily tight.

In the next section we experimentally test an algorithm that provides empirical evidence on the existence of a an algorithm that converges in polynomial time to a Nash equilibrium with constant cost-quality ratio.

3 Experimental Evaluation of the No-Profit Models

We conjecture that the Reachability game under the No-profit model (both Edge- and Path- payment policies) admits a Nash equilibrium and that there exists an equilibrium having a social cost within a constant factor of the cost of an optimal solution.

Some evidence in favor of the conjecture has been obtained by our experimental evaluations. We have tested the behavior of the algorithm described in Fig. 1. Procedure `findNE` takes as inputs the set of stations S and the broadcast source $s \in S$. As a first step, it computes a directed minimum spanning tree of S rooted at s and having arcs oriented towards the leaves (the MST-based algorithm mentioned in the introduction). Then, every station, in turn, tries to decrease the amount of its payments. This last step continues till a Nash equilibrium is found. Notice that, the `findNE` algorithm can be seen as a "simulation" of a distributed protocol for the construction of a broadcast range assignment by selfish stations that adjust a solution computed by the well known MST-based algorithm.

We have applied the algorithm `findNE` to two different kinds of instances: random instances and mobility instances. For the first ones, experiments have been carried out for several sizes n of the instances (between 10 and $2,000$) and for each n, one thousand instances have been randomly generated according to the uniform distribution. For the second ones, the instances have been generated by using a recently proposed mobility model whose main objective is to take into account the existence of obstacles and of pathways [11]. This mobility model tries to simulate this behavior as follows. Given a set of polygonal obstacles, it first computes the Voronoi diagram determined by the vertices of the

```
procedure findNE(S, s)
    T0 ← mst(S);
    compute T by rooting T0 at s and by orienting all its edges towards the leaves;
    for v ∈ S − {s} do
        pT(v) ← the sum of all payments due by v according to T
            and to the payment model;
        while T does not represent a Nash equilibrium do
            choose v ∈ S − {s};   m ← pT(v);   T2 ← T;
            for x ∈ S − {s} and x is not in the subtree of T rooted at v
                let u be the father of v in T;
                T1 ← E(T) − {(u, v)} ∪ {(x, v)}
                if pT1(v) < m then   m ← pT1(v);   T2 ← T1;
            if pT(v) < m then T ← T2;
        return T;
```

Fig. 1. The `findNE` algorithm

polygons: the edges of the diagram are the pathways that a mobile user has to follow. Subsequently, for each user, the source node and the destination node are randomly chosen among all the vertices of the Voronoi diagram. Finally, the user is moved along the minimal path (with respect to the diagram) between the source and the destination node with a randomly chosen speed. Once the user arrives at destination, a waiting time is randomly chosen: after this time, the movement process is repeated. By using this mobility model, experiments have been carried out for two obstacle scenarios and for different numbers n of users (between 10 and $2,000$): for each n, 100 instances have been generated according to the obstacle mobility model.

Remarkably, in all the experiments the algorithm `findNE` has been able to end in a Nash equilibrium in a very small number of rounds (a round is an iteration of the **while** loop).

Let $\langle S, s\rangle$ be an input of algorithm `findNE`. In what follows, the algorithm performance will be discussed by using the following parameters: the cost $\texttt{SC}(S)$ of the kick-off configuration (that is, the cost of the minimum spanning tree); the maximum cost $\texttt{WC}(S)$ between all configurations reached by the algorithm; the cost $\texttt{FC}(S)$ of the final configuration (that is the cost of the solution representing the Nash equilibrium); the executed number of rounds $\texttt{rnds}(S)$ before reaching the final state ($\texttt{rnds}(S) = 1$ if the kick-off configuration is a Nash equilibrium).

Convergence Speed. The necessary number of rounds for random instances is summarized in Table 2. This table shows that for the majority of the instances the convergence is within 6 rounds (1 round means that the starting solution is a Nash equilibrium). Moreover, only for a negligible number of instances the required rounds are in the interval $7 - 12$. No instance require more than 13 rounds.

The necessary number of rounds for mobility instances is summarized in Table 3. This table shows that for the majority of the instances the convergence is within 5 rounds. Notice that there are some differences between the two considered scenarios.

Table 2. Percentage of instances that converge to the Nash equilibrium in a given step. The rows indicate the cardinality of the instances and the columns the number of steps. Each column is divided in two sub-columns: the ones labelled with e refer to the Edge-payment model and the ones labelled with p refer to the Path-payment model.

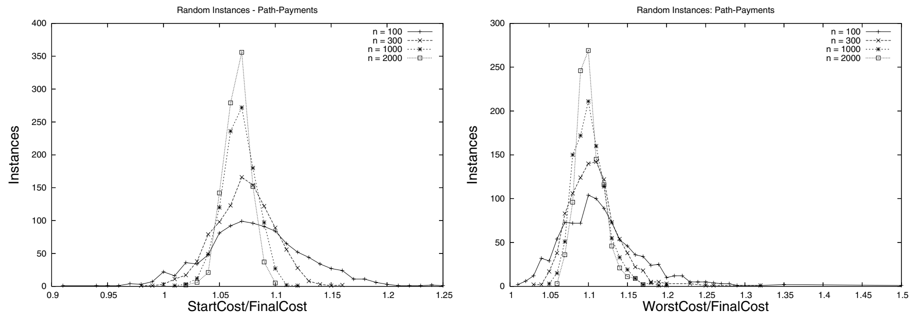
	1		2		3		4		5		6		7		...
n	e	p	e	p	e	p	e	p	e	p	e	p	e	p	...
10	40.9	12.0	50.9	69.5	7.5	16.8	0.6	1.5	0.0	0.1	0	0	0	0	...
100	0	0	46.4	5.2	48.9	65.9	4.6	25.4	0.1	3.3	0	0.2	0	0	...
200	0	0	24.1	0.1	67.9	50.5	7.8	40.8	0.2	7.2	0	1.3	0	0.1	...
300	0	0	10	0	77.2	33.9	12.3	54	0.4	9.6	0.1	1.7	0	0.4	...
400	0	0	4.4	0	79.6	23.8	15.5	55.4	0.5	16.5	0	3.7	0	0.5	...
500	0	0	3.1	0	76.9	15.5	19.1	61.6	0.9	17.8	0	3.5	0	1.3	...
1000	0	0	0.1	0	62.4	2.6	34.7	58.1	2.7	30.3	0.1	6.9	0	1.7	...
1500	0	0	0	0	50.9	1.3	46.3	41.8	2.7	45.3	0.1	10.4	0	0.9	...
2000	0	0	0	0	41.3	0.2	54	33.4	4.3	45.7	0.4	14.9	0	3.4	...

Table 3. Number of mobility instances that converge to the Nash equilibrium in a given round. The rows indicate the cardinality of the instances for the two scenarios and the columns the number of steps. Each column is divided in two sub-columns: the ones labelled with e refer to the Edge-payment model and the ones labelled with p refer to the Path-payment model. The rows are divided in two subrows, the first one refers to the first scenario, the second one to the second scenario.

		1		2		3		4		5		6		7		8	
n		e	p	e	p	e	p	e	p	e	p	e	p	e	p	e	p
10	Scen. 1	45	15	50	66	5	19	0	0	0	0	0	0	0	0	0	0
	Scen. 2	44	12	50	72	5	15	1	1	0	0	0	0	0	0	0	0
100	Scen. 1	1	0	75	42	20	50	4	8	0	0	0	0	0	0	0	0
	Scen. 2	1	0	72	8	26	67	1	20	0	4	0	1	0	0	0	0
200	Scen. 1	0	0	65	39	28	55	6	5	1	1	0	0	0	0	0	0
	Scen. 2	1	0	61	4	33	65	5	27	0	3	0	1	0	0	0	0
300	Scen. 1	0	0	70	37	25	58	3	5	2	0	0	0	0	0	0	0
	Scen. 2	0	0	65	4	28	64	6	25	1	7	0	0	0	0	0	0
400	Scen. 1	0	0	67	29	27	56	6	14	0	1	0	0	0	0	0	0
	Scen. 2	0	0	60	1	35	55	4	39	1	4	0	1	0	0	0	0
500	Scen. 1	0	0	93	22	7	64	0	13	0	1	0	0	0	0	0	0
	Scen. 2	0	0	53	1	46	57	1	35	0	7	0	0	0	0	0	0
1000	Scen. 1	0	0	69	28	23	66	8	5	0	0	0	1	0	0	0	0
	Scen. 2	0	0	88	0	12	51	0	39	0	9	0	0	0	0	0	1
1500	Scen. 1	0	0	91	20	7	76	1	4	1	0	0	0	0	0	0	0
	Scen. 2	0	0	66	1	33	45	1	41	0	13	0	0	0	0	0	0
2000	Scen. 1	0	0	68	69	22	26	8	5	2	0	0	0	0	0	0	0
	Scen. 2	0	0	3	0	56	1	41	70	0	25	0	3	0	1	0	0

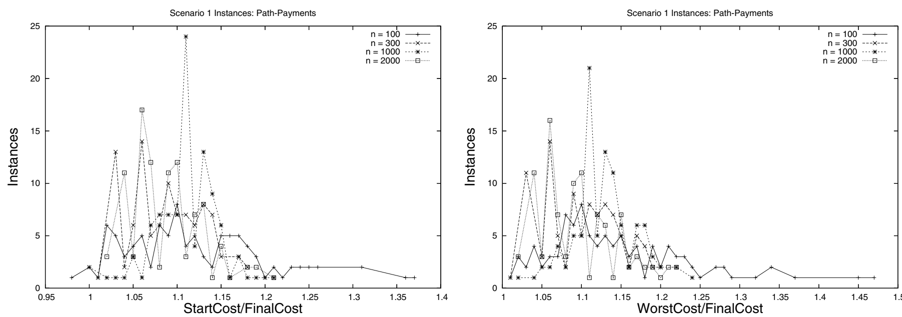
Quality of the Solutions. From the experiments we observe that, in the Edge-payment model $\texttt{SC}(S) = \texttt{WC}(S)$ for all the tested instances S.

The next question is, how far could be the social cost of the Nash equilibrium from the social cost of the optimum? Since the minimum spanning tree is a



(a) **(b)**

Fig. 2. Number of instances S with the same value SC(S)/FC(S) (a) and WC(S)/FC(S) (b) for random instances in the Path-payment model



(a) **(b)**

Fig. 3. Number of instances S with the same value SC(S)/FC(S) (a) and WC(S)/FC(S) (b) for mobility (scenario 1) instances in the Path-payment model

constant approximation of the optimum, we can compute the ratio between the cost of the minimum spanning tree (that is the starting solution) with the cost of the equilibrium. Notice that, due to the approximation property of the minimum spanning tree solution, this ratio is related with the cost-quality ratio.

In Fig. 2 it is shown the trend of the cost-quality ratio of the configurations generated by the `findNE` algorithm for the Path-payment model. In particular, Fig. 2.(a) shows the number of instance S with the same ratio SC(S)/FC(S) and Fig. 2.(b) shows the number of instance S with the same ratio WC(S)/FC(S) for the Path-payment model.

For the Edge-payment model, the ratio SC(S)/FC(S) has a similar trend to that in Fig. 2.(a). Notice that SC(S) = WC(S) in the Edge-payment model.

We then conjecture that the Nash equilibrium created by the `findNE` algorithm is a constant approximation of the optimal solution and the intermediate configurations are feasible solutions whose cost is an approximation of the optimal cost.

In Fig. 3 it is shown the number of instances S that have the same value SC(S)/FC(S) and WC(S)/FC(S) for mobility instances in the Path-payment model.

Also these results seem to support our conjecture that the Nash equilibria found by the `findNE` algorithm are constant approximations of optimal solutions. We obtain similar results for the other scenario and for the Edge-payment model.

4 Conclusions

In this paper we have studied the broadcast problem in the case of selfishly constructed ad-hoc networks. We used different non-cooperative game models depending on whether stations do not use payments, payments determine uniquely the existence of wireless links and agents can have profit. We have then considered two different payment policies, that is, Edge-payment and Path-payment, and we have proved the existence of a good (approximated) Nash equilibrium in the case of Path-payment and in the case of profit Edge-payment. Finally, we have given strong experimental evidence for the following conjecture (which is also the main problem left open by this paper): in the case of no-profit Edge-payment, there exists a polynomial time computable approximated Nash equilibrium that is an approximation of the optimal solution.

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