Exercises for the lecture

Best-response mechanisms

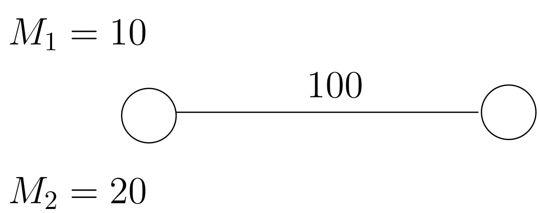
(with an application to BGP)

Exercise 1 Two players want to send data through a link of a certain capacity C. Each player can decide at which rate s_i to send its data (this is the **strategy** of player i), while the channel policy (if capacity is exceeded some part of data is dropped) assigns some actual rate a_i to each player (this amount is the **payoff** of player i). The available strategies for player i are $S_i = [0, M_i]$. The channel policy is **different** from the one discussed during the lecture and is as follows:[1]

$$a_1 := \min\{C, s_1\} \quad \text{and} \quad a_2 := \min\{C - a_1, r_2\}$$

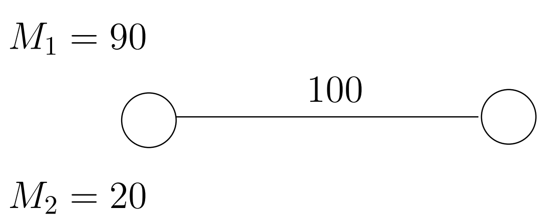
Prove the following things:

1. Look at this situation:

   Prove that this game is NBR-solvable (show the elimination sequence of Definition 4 of the lecture notes).

2. Now look at this situation (we've changed M_1):

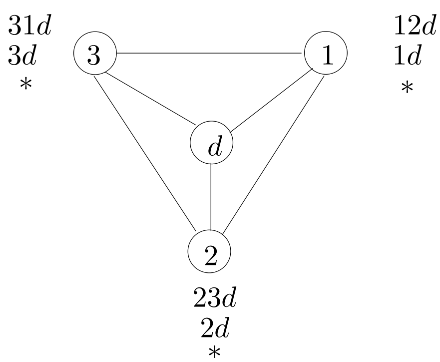
   Prove that also this game is NBR-solvable.

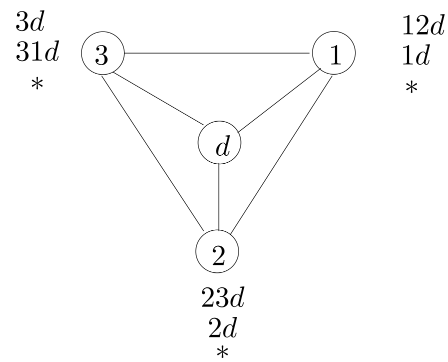
3. Combine the two proofs into a "general result", that is, show that for every C, and every for M_1 and M_2, the resulting game is NBR-solvable.

[1]Intuitively, we first satisfy player 1 and then give player 2 what is left.

4. Reconsider all three items above and prove that the game(s) is NBR-solvable **with clear outcome** (show the elimination sequence that satisfies Definition 7 of the lecture notes).

Exercise 2 Consider the network and the preferences of the nodes over the paths in these two pictures:





where $*$ denotes any path connecting the node to d but different from the top two. The utilities correspond to the rank: top-ranked path has utility 3, second-top-ranked has utility 2, $*$ has utility 1, and the utility is 0 if the node does not reach d (for example, if 3 transmits to 1 and 1 transmits to 3).

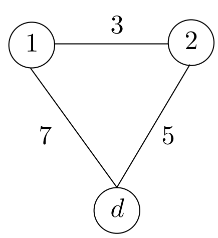
Prove the following:

1. For the left instance, there is no pure Nash equilibrium.

2. For the right instance, this is a NBR-solvable game.

Hint: As for point 2 show an elimination sequence of never best-response strategies satisfying Definition 4 in the lecture notes.

Exercise 3 Consider the following simple network



where the number on the links denote delays of the link. Suppose that nodes care only about the lenght of their own path (the sum of the delays of the path used by their own traffic) and they prefer shorter paths over longer paths:

$$P \prec_i Q \quad \text{if and only if} \quad \mathit{length}(P) < \mathit{lenght}(Q)$$

Prove the following:

1. For the above network there cannot be a Dispute Wheel, even if we change the delays (show that for any three positive delays the Dispute Wheel is impossible).

2. Try to generalize the proof to arbitrary networks: Every network with strictly positive delays cannot contain a Dispute Wheel.

3. Assume you have proved the previous item. Explain why this would imply that BGP (best-response) converges when nodes' preferences are base on the length of the paths as described above.